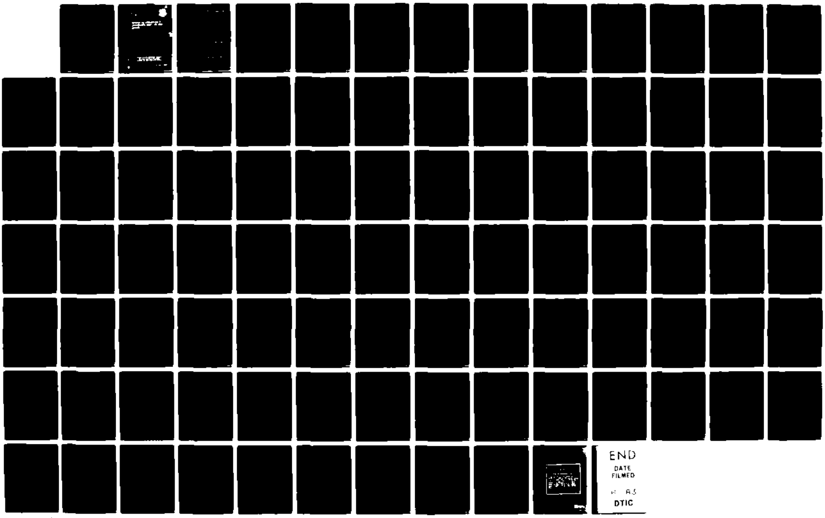
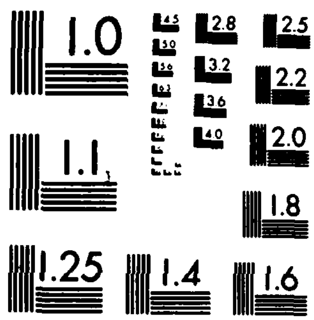
AD-A128 436   SCATTERING AND DEPOLARIZATION OF ELECTRO-MAGNETIC WAVES   1/1
- FULL WAVE SOLUTIONS(U) NEBRASKA UNIV LINCOLN  E BAHAR
MAR 83 RADC-TR-82-336 F19628-81-K-0025

UNCLASSIFIED   F/G 20/14   NL






MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A128436

# SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES - FULL WAVE SOLUTIONS

University of Nebraska-Lincoln

Ezekiel Bahar



APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

APPROVED: [signature]

K.V.N. RAO
Project Engineer

APPROVED: [signature]

ALLAN C. SCHELL, Chief
Electromagnetic Sciences Division

FOR THE COMMANDER: [signature]

JOHN P. [illegible]
Acting Chief, Plans Office

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | | |
|---|---|---|
| 1. REPORT NUMBER: RADC-TR-82-336 | 2. GOVT ACCESSION NO.: AD-A128 436 | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle): SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES - FULL WAVE SOLUTIONS | | 5. TYPE OF REPORT & PERIOD COVERED: Interim Report 1 Mar 81 - 28 Feb 82 |
| | | 6. PERFORMING ORG. REPORT NUMBER: N/A |
| 7. AUTHOR(s): Ezekiel Bahar | | 8. CONTRACT OR GRANT NUMBER(s): F19628-81-K-0025 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of Nebraska - Lincoln, 414 Administration Bldg, 14th and R Sts, Lincoln NE 01731 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2305J442 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS: Rome Air Development Center (EECT), Hanscom AFB MA 01731 | | 12. REPORT DATE: March 1983 |
| | | 13. NUMBER OF PAGES: 94 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Same | | 15. SECURITY CLASS. (of this report): UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: N/A |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

RADC Project Engineer: K.V.N. Rao (EECT)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electromagnetic Scattering and Depolarization Composite Rough Surfaces
Physical Optics
Bragg Scattering
Full Wave Approach

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report consists of three principal parts (Section 2, 3, and 4). The purpose of the research reported in Section 2 is to resolve the apparent discrepancies between different Physical Optics solutions for rough surface scattering. It is also shown in this section that the appearance of the so-called "edge effect" is due to premature truncation of a closed surface integral.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

In Section 3, full wave solutions are developed for the depolarization of the scattered radiation fields by conducting objects of irregular shape above rough land and sea. The research reported in this section deals primarily with the deterministic problem.

In Section 4 the full wave approach, which accounts for both Bragg scattering and specular point scattering in a self-consistent manner, is employed to evaluate the scattering cross sections for composite models of rough surfaces. It is shown that the full wave solutions for the scattering cross sections can be expressed as a weighted sum of two cross sections. The like polarized cross sections which are associated with the large scale surface are independent of frequency and polarization. The cross section which is associated with the small scale surface is dependent on frequency and polarization.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd.)

## 1.0 INTRODUCTION

The recent impetus to produce rigorous solutions to more realistic models of pertinent propagation problems over a very wide frequency range has generated the need to derive full wave solutions to problems of radio wave propagation in dispersive, inhomogeneous, anisotropic and dissipative media with irregular boundaries. The considerable growth in civil and military interest in the development of more reliable systems for communication, detection, navigation and positioning, the potential for developing radio wave methods for remote sensing and the need to develop secure hardened communication systems have contributed much to this renewed interest. These developments have been paralleled by remarkable advances that have been made in the availability of high powered, very low frequency electromagnetic sources as well as the availability of transmitters operating at optical frequencies. The ready access to large, versatile digital computers has made it possible to employ the full wave approach to obtain numerical solutions to a wide class of important problems which have hitherto been either ignored or over-idealized in order to reduce them to tractable problems.

To perform the full wave analyses, it is necessary to develop generalized field transforms that provide the basis for the complete expansions for the electromagnetic fields in irregular multilayered structures with varying thickness and electromagnetic parameters. These complete expansions consist of the vertically and horizontally polarized radiation fields, lateral waves and guided surface waves. The generalized field transforms are used to reduce Maxwell's equations, in conjunction with the associated exact boundary conditions for the

electromagnetic fields, into sets of first order coupled differential equations for the forward and backward traveling wave amplitudes.

The full wave solutions, that have been derived for the scattered radiation fields from rough surfaces with arbitrary slope and electromagnetic parameters, bridge the wide gap that exists between the perturbational solutions for rough surfaces with small slopes and the Physical Optics solutions.

Computer programs are being developed to numerically evaluate the scattering cross sections. Thus the solutions obtained using the full wave approach can also be used by engineers who are not necessarily familiar with the analytical techniques used in the analysis.

## 1.1 Summary of Research

In his comprehensive review of theoretical treatments of scattering of electromagnetic waves by rough surfaces, Valenzuela (1968) states that scattering models have been developed "that make it possible to apply available theories to surfaces that cannot be treated purely by perturbation and Physical Optics..." However, Valenzuela notes that, "these composite models are mostly based on physical considerations and are able to explain features in radar cross-section data from the sea that no theory can."

For instance, these composite models are used to show that backscatter at near normal incidence is primarily due to specular point scattering while backscatter at near grazing angles is primarily due to Bragg scattering. Thus, for perfectly conducting rough surfaces the backscatter cross section for near normal incidence is independent of frequency and polarization while at near grazing angles it is dependent on both frequency and polarization.

Brown (1978, 1980) recently employed a combination of Burrows perturbation theory (1967) and Physical Optics theory (Beckmann and Spizzichino, 1963; Beckmann, 1968) to obtain a solution for the back-scatter cross sections for perfectly conducting rough surfaces in terms of a sum of two cross sections. The first term in his solution is the specular point backscatter cross section associated with the large scale (filtered) surface height and the second term is the Bragg scatter cross section associated with the small scale surface height. Thus in his work it is necessary to decompose (i.e., spectrally filter) the rough surface. To this end, Brown's specification of the wavenumber $k_d$, where spectral splitting is assumed to occur, is based entirely upon the characteristics of the small scale structure (1978). However, in the approaches of Hagfors (1966) and Tyler (1976), the specification of $k_d$ is assumed to be based on the characteristics of the large scale surface. Furthermore Brown's results for the total backscatter cross sections are critically dependent on the specific value chosen for $k_d$ (Brown, 1978).

In this work the full wave approach, which accounts for both Bragg scattering and specular point scattering in a self-consistent manner, is employed to evaluate the scattering cross sections for composite models of rough surfaces (Bahar, 1981a,b). It is shown that the full wave solutions for the scattering cross sections can be expressed as a <u>weighted</u> sum of two cross sections. The like polarized cross sections which are associated with the large scale surface are independent of frequency and polarization. The cross section which is associated with the small scale surface is dependent on frequency and polarization.

Furthermore the specification of $k_d$ is not based entirely on the characteristics of the small scale structure. To determine the versatility of the full wave approach, it is necessary to examine the sensitivity of the total scattering cross section to variations in the value of $k_d$ where spectral splitting is assumed to occur.

The purpose of the research reported in Section 2 is to resolve the apparent discrepancies between different Physical Optics solutions for rough surface scattering. It is also shown in this section that the appearance of the so-called "edge effect" in Beckmann's results (Beckmann and Spizzichino, 1963), is due to premature truncation of a closed surface integral.

In Section 3, full wave solutions are developed for the depolarization of the scattered radiation fields by conducting objects of irregular shape above rough land and sea. The research reported in this section deals primarily with the deterministic problem. In Section 4 these solutions are applied to random rough surfaces. It is shown that the full wave solution bridges the wide gap between the Physical Optics approach and the perturbation solutions.

We conclude this section with a summary of the principal elements of the full wave approach. The principal properties of the full wave solution and its relationships to earlier solutions of scattering problems are also summarized (Bahar, 1981c). This summary is also presented schematically in Figs. 1.1 and 1.2. The reader of this report who is not familiar with the full wave approach will find this summary useful even though the details of the full wave method have been reported earlier (Bahar, 1973a,b, 1974, 1981).

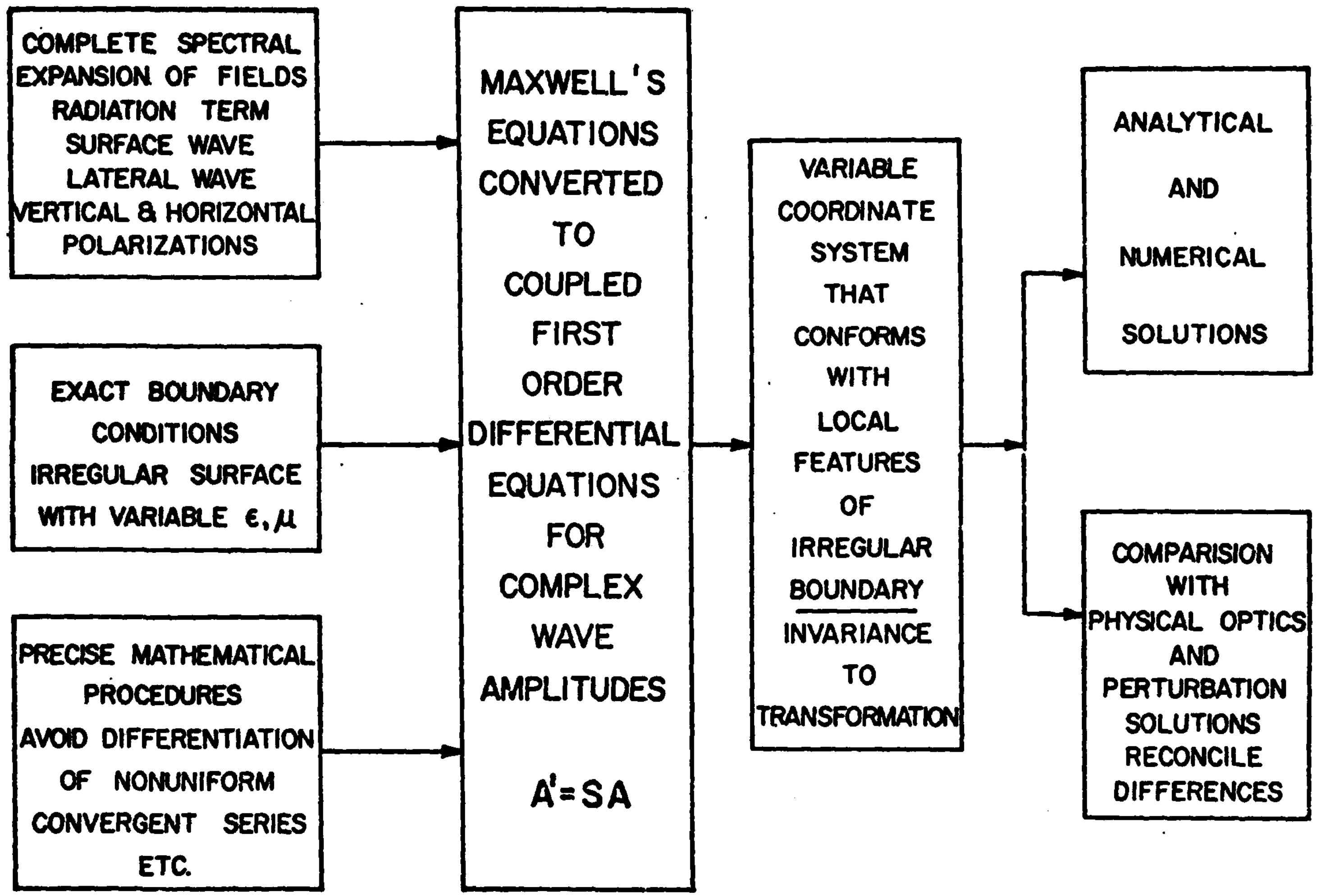



Figure 1.1 Principal Elements of Full Wave Approach.

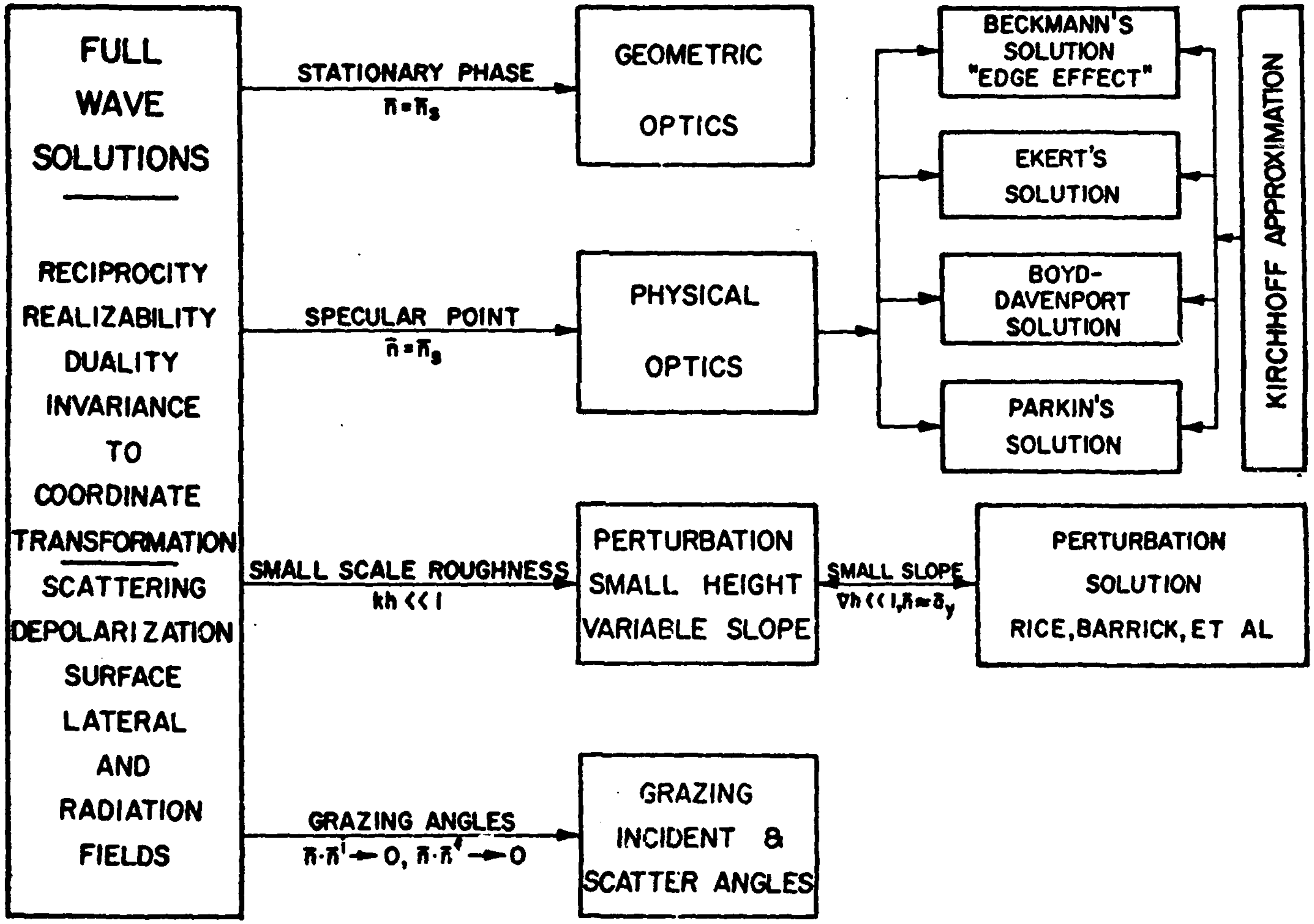



Figure 1.2 Principal properties of Full Wave Approach.

(a) The electromagnetic fields are expressed in terms of complete expansions of vertically and horizontally polarized waves. These include the radiation fields, the lateral waves and the surface waves (Bahar, 1973a,b; 1974).

(b) Exact boundary conditions are imposed at the irregular surface.

(c) Using the orthogonal properties of the basis functions appearing in the complete expansions of the fields, Maxwell's equations are integrated over the transverse plane (y,z) (Bahar, 1973a,b; 1974). Green's theorems are used to avoid term-by-term differentiation of the field expansions.

(d) Maxwell's equations for the electromagnetic fields are converted into coupled first order ordinary differential equations for the forward and backward traveling wave amplitudes which are only functions of the variable x (Bahar, 1973a,b; 1974). (In view of the integration in the transverse plane (y,z) the telegraphists' equations are only functions of x). The coupled equations for the wave amplitudes are referred to the generalized telegraphists' equations (Bahar, 1981c).

(e) Second order iterative solutions for the radiation fields are obtained from the telegraphists' equations on neglecting multiple scattering from the rough surface.*

(f) A variable coordinate system that conforms with the local features of the irregular boundary is introduced and the resulting solutions for the scattered fields are shown to be invariant to coordinate transformations.

(g) The full wave solutions are also shown to satisfy the reciprocity realizability and duality relationships in electromagnetic theory.

---

*These second order iterative solutions account for wave scattering in arbitrary directions.

(h) The full wave approach not only accounts for scattering and depolarization of the radiation fields but also accounts for coupling between the surface waves , the lateral waves and the radiation fields.

The versatility of the full wave approach is demonstrated by determining its relationship to earlier solutions. Thus on using a stationary phase approach to evaluate the integrals for the scattered fields, the full wave approach is shown to reduce to the geometric optics solutions (Bahar, 1981c). On the other hand, if the vector $\bar{n}$ normal to the rough surface is replaced by its value at the specular points $\bar{n}_s$, the full wave expressions for the scattered fields are shown to reduce to the Physical Optics solutions. Thus the Physical Optics approach is valid only if the contributions to the scattered fields come primarily from specular points on the rough surface. Details of this aspect of the research are given in Section 3.6.

In a survey of the technical literature one finds several different forms of Physical Optics solutions. The discrepancies between the different Physical Optics solutions and the appearance of the so-called "edge effect" have been shown to be the result of premature truncation of the closed surface integrals. Details of this research are reported in Section 2.

If one assumes that the scale and the slopes of the rough surface are small, it is shown that the full wave solutions reduce to the perturbation solutions. Details of this aspect of the research are given in Section 3.

The Physical Optics solutions for the backscattered fields become singular for near grazing angles. Thus in this case, even if the rough surface satisfies the radii of curvature criteria (associated with the Kirchhoff approximations of the surface fields), the Physical Optics

solutions cannot be used. This is because of near grazing angles, the principal contributions to the backscattered fields do not come from specular points of the rough surface. (In this case specular points, if they existed, would be on vertical portions of the rough surface). It is shown that the full wave solutions for the backscattered fields remain valid as one approaches grazing angles (Bahar, 1982).

## 1.2 Interim Technical Reports

Preprints of the following manuscripts were submitted to the Contract Monitor and published as a U.S. Air Force Interim Technical Report, RADC-TR-81-204, August 1981.

Full Wave Analysis for Scattering Cross Sections
Part 1 Random Rough Surfaces
Part 2 Composite Surfaces

## 1.3 References


1. Bahar, E., "Depolarization of Electromagnetic Waves Excited by Distribution of Electric and Magnetic Sources in Inhomogeneous Multilayered Structures of Arbitrarily Varying Thickness-Generalized Field Transforms," Journal of Mathematical Physics, 14(11), pp. 1502-1509, 1973.
2. Bahar, E., "Depolarization of Electromagnetic Waves Excited by Distributions of Electric and Magnetic Sources in Inhomogeneous Multilayered Structures of Arbitrarily Varying Thickness-Full Wave Solutions," Journal of Mathematical Physics, 14(11), pp. 1510-1515, 1973.

3. Bahar, E., "Depolarization in Nonuniform Multilayered Structures--Full Wave Solutions," Journal of Mathematical Physics, 15(2), pp. 202-208, 1974.

4. Bahar, E., "Scattering Cross Section from Random Rough Surfaces--Full Wave Analysis," Radio Science, 16(3), pp. 331-341, 1981a.

5. Bahar, E., "Scattering Cross Sections for Composite Random Surfaces--Full Wave Analysis," Radio Science, 16(6), pp. 1327-1335, 1981b.

6. Bahar, E., "Full Wave Solutions for the Depolarization of the Scattered Radiation Fields by Rough Surfaces of Arbitrary Slope," IEEE Transactions on Antennas and Propagation, AP-29(3), pp. 443-454, 1981.

7. Bahar, E., "Scattering and Depolarization by Rough Surfaces Near Grazing Angles--Full Wave Solutions," IEEE Transactions on Antennas and Propagation, AP-30(4), pp. 712-719, 1982.

8. Beckmann, P., The Depolarization of Electromagnetic Waves, Golem Press, Boulder, Colorado, 1968.

9. Beckmann, P., and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, MacMillan, New York, 1963.

10. Brown, G. S., "Backscattering from a Gaussian-Distributed Perfectly Conducting Rough Surface," IEEE Transactions on Antennas and Propagation, AP-26(3), pp. 472-482, 1978.

11. Brown, G. S., "Correction to Backscattering from a Gaussian-Distributed Perfectly Conducting Rough Surface," IEEE Transactions on Antennas and Propagation, AP-28(6), pp. 943-946, 1980.

12. Burrows, M. L., "On the Composite Model for Rough Surface Scattering," IEEE Transactions on Antennas and Propagation, AP-21(2), pp. 241-243. 1967.

13. Hagfors, T., "Relationship of Geometric Optics and Autocorrelation Approach to the Analysis of Lunar and Planetary Radar," Journal of Geophysical Research, 71, pp. 379-383, 1966.

14. Tyler, G. L., "Wavelength Dependence in Radio-Wave Scattering and Specular Point Theory," Radio Science, 11(1), pp. 83-91, 1976.

15. Valenzuela, G. R., "Scattering of Electromagnetic Waves from a Tilted Slightly Rough Surface," Radio Science, 3(11), pp. 1051-1066, 1968.

# 2.0 RESOLUTION OF THE DISCREPANCIES BETWEEN DIFFERENT PHYSICAL OPTICS SOLUTIONS FOR ROUGH SURFACE SCATTERING

## 2.1 Background

The purpose of the analysis presented in this section is to resolve the discrepancies between the different high frequency, Physical Optics expressions for the scattered field derived by several researchers in the field. The analysis also resolves the questions associated with the "edge effect" that appears in some of the earlier solutions. It is shown that the Physical Optics approach is only applicable for specular point scattering and that the so-called "edge effect" which appears in earlier derivations of the Physical Optics solution for rough surface scattering is a result of premature truncation of the closed surface integral expression for the scattered fields. Therefore, this term must be suppressed even when it is not very small compared to the scattered field in the off specular direction. Since the Kirchhoff approximations for the surface fields are used in the Physical Optics approach, it cannot account for wave diffraction by edges.

The Physical Optics solution derived here for arbitrary source excitation is shown to satisfy reciprocity and realizability relationships in electromagnetic theory. The integrand in the integral expression for the scattered field is identified with the specific reflectance (per unit area) of the rough surface. Although the scalar acoustic problem is considered here in detail, the results are also applicable to electromagnetic scattering.

The high frequency approximation of the full wave solution for the scattered field is in agreement with the Physical Optics solution derived here.

## 2.2 Discussion

Applying Green's theorem to problems of rough surface scattering and using the Kirchhoff approximations for the fields on the surface, Physical Optics (high frequency) solutions for the scattered fields have been derived in terms of an integral over the rough surface (Beckmann and Spizzichino, 1963). On performing integration by parts, a boundary term which does not depend upon the shape of the rough surface appears in the solution. When the scattering surface is large (in terms of wavelengths), this term which is identified by Beckmann as an "edge effect" is considered to be very small compared to the remaining surface integral and therefore negligible.

Several researchers in the field have clearly demonstrated however, that in general the so-called "edge effect" is not necessarily very small in comparison with the remaining surface integral. It has been suggested that the "edge effect" . . . . "may become important when considering scattering in directions off specular" (Fung, Moore and Parkins, 1965). Wetzel (1966) states that Beckmann's argument that the integrated terms (edge effect) can be discarded "leads to an incorrect result for the flat surface limit and suggests a scheme to rectify the discrepancy by retaining the integrated terms." Furthermore, Valenzuela (1968) finds "that Beckmann's approximation in which he neglected the 'Edge effect' leads to incorrect predictions at angles away from normal

incidence" and derives solutions "without neglecting the 'edge effect' terms." However, it is shown (Section 2.4) that when the term identified with the "edge effect" is included, the Physical Optics solution (Beckmann and Spizzichino, 1963) does not satisfy the reciprocity relationship in electromagnetic theory.

In addition, a peculiar non-vanishing term appears in the expression for the scattered field even as the reflection coefficient for the rough surface is set equal to zero. This unacceptable result becomes more pronounced in the off specular direction.

To compound the problems one encounters with the so-called "edge effect," Wetzel (1975) points out that the three formulations for the scattered field derived by Eckhart (1953), Parkins (1967), Boyd-Deavenport (1973) "give three different results for the scattered field." What is not so obvious (according to Wetzel) is "why this should be." Wetzel goes on to suggest that "These inconsistent results, . . . must somehow be associated with the use of the Kirchhoff approximation in the scalar Helemholtz integral." All three formulations cited above are based on the Kirchhoff approximations and the discrepancies are obvious even before the integration by parts and the appearance of the term associated with the "edge effect."

Several questions therefore need to be answered. Should the so-called "edge effect" be included in the solution if it is not very small compared to the scattered field? If the "edge effect" does "become important" and is included in the solution, how can one account for the fact that the resulting Physical Optics solution does not satisfy reciprocity and does not vanish as the reflection coefficient becomes vanishingly small? How can one resolve the discrepancies in the formulations by Eckart (1953), Parkins (1967) and Boyd-Deavenport (1973)

even though all apply Kirchhoff approximations and none involve the so-called "edge effect"?

To this end, in Section 2.3, the general form of the Physical Optics solution for the scattered fields is presented in terms of a closed surface integral and an arbitrary source distribution (spherical waves rather than plane wave excitation) is assumed. In Section 2.4 the limiting forms of the Physical Optics solutions with and without the "edge effect" are derived for plane wave excitations. Realizability and reciprocity relationships in electromagnetic theory are examined and the limiting case in which the reflection coefficient vanishes is also considered.

In Section 2.5 it is shown that the discrepancies between the different formulations for the scattered field and the problems that arise with the so-called "edge effect" in the Physical Optics solution is due to the premature truncation of the closed surface integral encountered in the application of Green's theorem. On applying the divergence theorem (in two dimensions), the term associated with the "edge effect" is shown to vanish identically for all scatter directions. The integrand in the final expression derived here for the Physical Optics scattered field is identified with the specific reflectance (per unit area) of the rough surface. Furthermore, the integrand is not proportional to the gradient of the rough surface and is therefore not undefined at edges. The integration can be performed only over those portions of the rough surface with a non-vanishing reflection coefficient. Shadowing effects are considered and the distinction between the scattered field, the reflected field and the shadow forming field are taken into account (Morse and Feshbach, 1953).

The final form for the Physical Optics scattered field satisfies the reciprocity and realizability relationships. It vanishes as the reflection coefficient vanishes at the rough surface for all incident and scatter angles. Using the Physical Optics approach, the term associated with the "edge effect" must be suppressed even for scattering far off the specular direction. The Physical Optics solution which employs a Kirchhoff approximation for the surface fields cannot account for wave diffraction by edges.

In Appendix (2.A), it is shown that on following the analytical procedures of Section 2.5, the Eckart, Parkins and Boyd-Deavenport solutions are in agreement provided the integration is over the closed surface $A_o$. Furthermore, the integrand in all the above Physical Optics solutions (which employ the Kirchhoff approximations) are shown to be equal at the specular (stationary phase) points.

While the "Helmholtz Integral Equation" (2.10) (associated with the scalar problem) is considered here in detail, it is interesting to note that similar difficulties arise when the "Stratton-Chu Integral Equation" (associated with the corresponding vector problem) is applied to surfaces that are not closed (Barrick, 1965).

## 2.3 Formulation of the Problem

Consider the solution to the scalar acoustic potential $\psi(\bar{r})$ that satisfies the inhomogeneous wave (Helmholtz) equation

$$(\nabla^2 + k^2)\psi(\bar{r}) = -4\pi\, s(\bar{r}) \tag{2.1}$$

in which $s(\bar{r})$ is proportional to the acoustic source density and the acoustic wave number is

$$k = \omega/c \quad , \tag{2.2}$$

in which c, the acoustic velocity, is related to the medium density $\rho$ and elasticity $\lambda$ and acoustic impedance Z through the equations (Brekhovskikh,1960)

$$c = \sqrt{\frac{\lambda}{\rho}} \quad , \; Z = \rho c = \sqrt{\rho\lambda} \tag{2.3}$$

The acoustic pressure P and particle velocity $\bar{V}$ are

$$\bar{V} = -\text{grad}\ \psi \quad , \quad P = -i\omega\rho\ \psi \tag{2.4}$$

and the boundary conditions at an interface between two media characterized by $c,\rho$ and $c_1,\rho_1$ respectively are

$$P(\bar{r}_s) = P_1(\bar{r}_s) \text{ and } \bar{n}\cdot\bar{V}(\bar{r}_s) = \bar{n}\cdot\bar{V}_1(\bar{r}_s) \tag{2.5}$$

in which $\bar{n}$ is the unit vector normal to the interface. Thus at $\bar{r} = \bar{r}_s$

$$\rho\psi = \rho_1\psi_1 \tag{2.6a}$$

and

$$\bar{n}\cdot\nabla\psi = \bar{n}\cdot\nabla\psi_1 \text{ or } \frac{\partial\psi}{\partial n} = \frac{\partial\psi_1}{\partial n} \tag{2.6b}$$

To facilitate the solution to (2.1) consider also the Green's function $G(\bar{r}|\bar{r}_o)$ that satisfies the equation

$$(\nabla^2 + k^2)G(\bar{r}|\bar{r}_o) = -4\pi\delta(\bar{r}-\bar{r}_o) \tag{2.7}$$

in which $\delta(\bar{r}-\bar{r}_o)$ is the Dirac delta function. For an infinite medium characterized by parameters $\rho,c$ (Morse and Feshbach,1953)

$$G(\bar{r}|\bar{r}_o) = G(\bar{r}_o|\bar{r}) = e^{ik|\bar{r}-\bar{r}_o|}\left|\frac{1}{\bar{r}-\bar{r}_o}\right| \tag{2.8}$$

in which an $\exp(-i\omega t)$ time dependence is assumed. Applying Green's second theorem to the volume $V_o$ bounded by the closed surface $A_o$ separating the two media, $\rho,c$ and $\rho_1,c_1$ one gets

$$\int(\psi\nabla^2 G-G\nabla^2\psi)dV_o = \oint [\psi\nabla\ G-G\nabla\ \psi]\cdot\overline{dA}_o \tag{2.9}$$

Thus (Morse and Feshbach,1953),

$$\psi(\bar{r}) = \int s(\bar{r}_o)G(\bar{r}|\bar{r}_o)dV_o + \frac{1}{4\pi}\oint[\psi(\bar{r}_o^s)\nabla_o G(\bar{r}|\bar{r}_o^s) - -G(\bar{r}|\bar{r}_o^s)\nabla_o\psi(\bar{r}_o^s)]\cdot\bar{n}\, dA_o \quad (2.10)$$

in which both the source and observation point are in the volume $V_o$ and $\overline{dA}_o = -\bar{n}dA_o$ where $\bar{n}$ is the unit normal to the surface $A_o$ pointing towards the medium in which the source and observation points are located (see Fig. 2.1). If medium $\rho,c$ is infinite ($\rho=\rho_1$, $c=c_1$) the surface integral vanishes and $\psi(\bar{r})$ reduces to the incident unperturbed field $\psi^i(\bar{r})$ (Morse and Feshbach,1953). Thus

$$\psi^i(\bar{r}) = \int s(\bar{r}_o)\frac{e^{ik|\bar{r}-\bar{r}_o|}}{|\bar{r}-\bar{r}_o|}\, dV_o \quad (2.11)$$

If the source $s(\bar{r}_o)$ is in the vicinity of the origin

$$|\bar{r}-\bar{r}_o| \simeq r^i-\bar{r}_o\cdot\bar{a}_r \ , \ \bar{a}_r = \bar{r}/r = \bar{n}^i \ , \ |\bar{r}| = r^i \ . \quad (2.12)$$

the far field expression for $\psi^i$ is

$$\psi^i(\bar{r}) \simeq \frac{e^{ikr^i}}{r^i}\int s(\bar{r}_o)e^{-ik\bar{r}_o\cdot\bar{n}^i}\, dV_o = g(\bar{n}^i)\,\frac{e^{ikr^i}}{r^i} \quad (2.13)$$

in which the gain function $g(\bar{n}^i)$ depends upon the direction of the unit vector $\bar{n}^i(\theta,\phi)$ pointing from the source (in the vicinity of the origin) to the point at $\bar{r}$ (Jordan and Balmain,1968). Since $g(\bar{n}^i)$ is not a function of $r^i$, the distance from the source to the field point, the far field approximation for the incident velocity vector $\bar{V}^i(\bar{r})$, (2.4), is given by

$$\bar{V}^i = -\nabla\psi^i(\bar{r}) = -(ik - \frac{1}{r^i})\bar{n}^i\psi^i(\bar{r}) \simeq -ik\bar{n}^i\psi^i(\bar{r}) \quad (2.14a)$$

Thus for the far field

$$\bar{V}^i = ik\,\bar{n}^i\,P^i/i\omega\rho = \bar{n}^iP^i/Z \quad (2.14b)$$

in which Z is the acoustic impedance (2.3). Expressing the total acoustic potential $\psi$ as the sum of the incident (unperturbed) and scattered

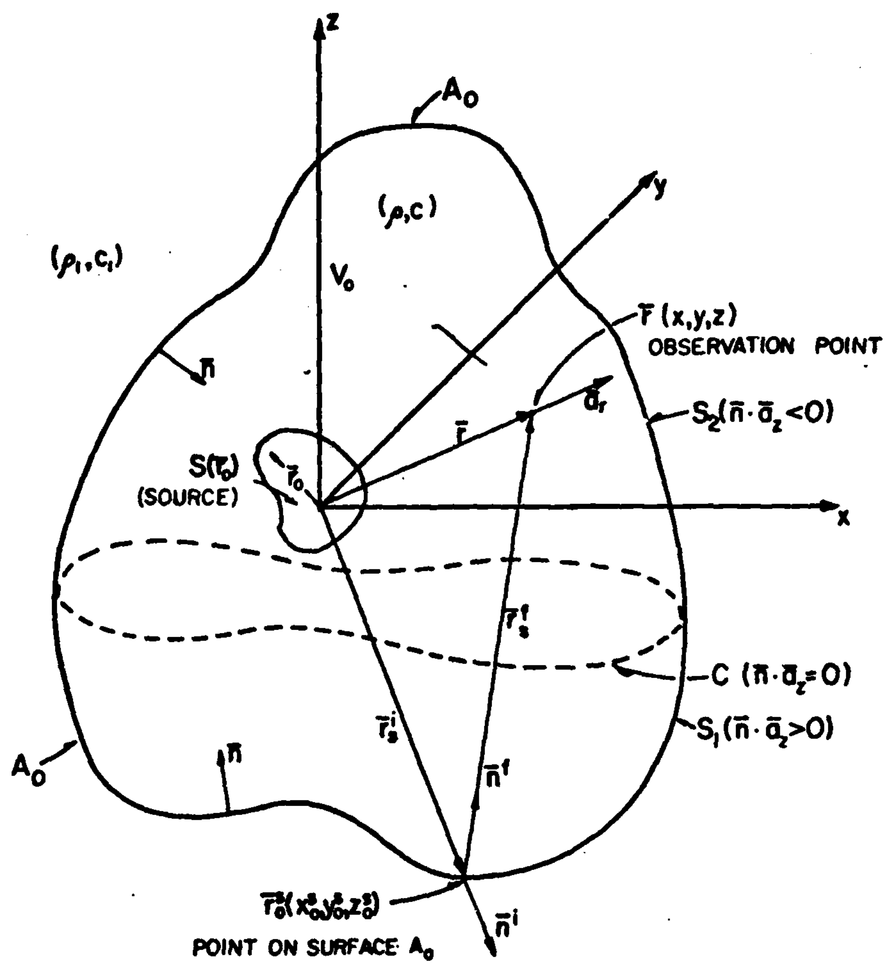



Figure 2.1 Source point, observation point and the boundary surface.

potential $\psi^i$ and $\psi^s$ respectively

$$\psi = \psi^i + \psi^s \tag{2.15}$$

equation (2.10) reduces to (Morse and Feshbach 1953)

$$\psi^s(\bar{r}) = \frac{1}{4\pi}\oint[\psi^s(\bar{r}_o^s)\nabla_o G(\bar{r}|\bar{r}_o^s) - G(\bar{r}|\bar{r}_o^s)\nabla_o\psi^s(\bar{r}_o^s)]\cdot\bar{n}dA_o \tag{2.16a}$$

and

$$0 = \frac{1}{4\pi}\oint[\psi^i(\bar{r}_o^s)\nabla_o G(\bar{r}|\bar{r}_o^s) - G(\bar{r}|\bar{r}_o^s)\nabla_o\psi^i(\bar{r}_o^s)]\cdot\bar{n}dA_o \tag{2.16b}$$

The Kirchhoff (Physical Optics) approximations for $\psi(\bar{r}_o^s)$ and $\nabla_o\psi(r_o^s)$ are

$$\psi(\bar{r}_o^s) = (1 + R)\psi^i(\bar{r}_o^s) \ , \ \nabla_o\psi(\bar{r}_o^s) = ik\bar{n}^i(1 - R)\psi^i(\bar{r}_o^s) \tag{2.17}$$

in which $R(-\bar{n}^i\cdot\bar{n})$ is the acoustic reflection coefficient at a flat interface between medium $\rho, c$ (where the incident wave originates) and medium $\rho_1, c_1$ (Brekhovskikh,1960; Ishimaru,1978). Substituting the far field approximation for $\nabla_o G(\bar{r}|\bar{r}_o^s)$ in (2.10) the Kirchhoff approximation for the scattered acoustic potential is

$$\begin{aligned}\psi^s(\bar{r}) &= -\frac{ik}{4\pi}\oint[(1+R)\bar{n}^f - (1-R)\bar{n}^i]\psi^i(\bar{r}_o^s)G(\bar{r}|\bar{r}_o^s)\cdot\bar{n}dA_o \\ &= \frac{i}{4\pi}\oint \bar{M}\cdot\bar{n}\ g(\bar{n}^i)\ \frac{\exp[ik(r_s^i+r_s^f)]}{r_s^i\ r_s^f}\,dA_o\end{aligned} \tag{2.18}$$

in which $r_s^i$ and $r_s^f$ are distances from a point on the surface $A_o$ and the source and observation points respectively,

and
$$\left.\begin{aligned}\bar{M} &= R\bar{v} - \bar{p} \ , \ \bar{p}=k(\bar{n}^i+\bar{n}^f) = p_x\bar{a}_x + p_y\bar{a}_y + p_z\bar{a}_z \\ v &= k(\bar{n}^i-\bar{n}^f) = v_x\bar{a}_x + v_y\bar{a}_y + v_z\bar{a}_z\end{aligned}\right\} \tag{2.19}$$

The results of the analysis for the non-depolarized scattered electromagnetic field is the same as (2.18) with the exception that the acoustic reflection coefficient is replaced by the Fresnel reflection coefficients for horizonatally and vertically polarized waves (Beckmann and Spizzichino,1963).

## 2.4 The Physical Optics Solution and the So-Called "Edge Effect"

In order to apply Green's theorem (2.9) to surfaces that are not closed, Beckmann and Spizzichino (1963) suggest that "one may regard the rough surface S as part of any closed surface S' of which only S has a non-vanishing reflection coefficient, the remaining integral over S'-S is then easily shown to vanish so that there is no need to introduce the surface S'".

Furthermore Beckmann assumes that the surface S is such that the distances $r_s^i$ and $r_s^f$ from the source and the observation points to points on the surface S are much larger than distances between two points on the rough surface. In this case it is convenient to define a new coordinate system $\bar{r}_s(x_s,y_s,z_s)$ associated with the surface S whose origin is shifted from the original coordinate system $\bar{r}(x,y,z)$ such that

$$\bar{r}(x,y,z) = \bar{r}_o^i + \bar{r}_s(x_s,y_s,z_s) \tag{2.20a}$$

The surface S is defined by

$$f(x_s,y_s,z_s) = z_s - \zeta(x_s,y_s) = 0, \; -X < x_s < X, -Y < y_s < Y \tag{2.20b}$$

In (2.20) $r_o^i$ is the constant vector joining the origins of the two coordinate systems and $z_s = 0$ is the mean plane of the surface S, thus

$$\int_s \zeta(x_s,y_s)dx_s dy_s = 0 \tag{2.21}$$

In this case for points on the surface S, $\bar{n}^i$ and $\bar{n}^f$ are constant and

$$r_s^i \simeq r_o^i + \bar{n}^i \cdot \bar{r}_s \qquad r_s^f \simeq r_o^f - \bar{n}^f \cdot \bar{r}_s \tag{2.22}$$

The distances from the source and observation point to the origin of the new coordinate system are $r_o^i$ and $r_o^f$ respectively. (see Fig. 2.2). Thus for points on the surface S, the incident field $\psi^i(\bar{r}_o^s)$ is

$$\psi^i(\bar{r}_o^s) = g(\bar{n}^i)\,\frac{e^{ikr_o^i}}{r_o^i}\,e^{ik\bar{n}^i\cdot\bar{r}_s} = E_o^i\,e^{ik\bar{n}^i\cdot\bar{r}_s} \tag{2.23}$$

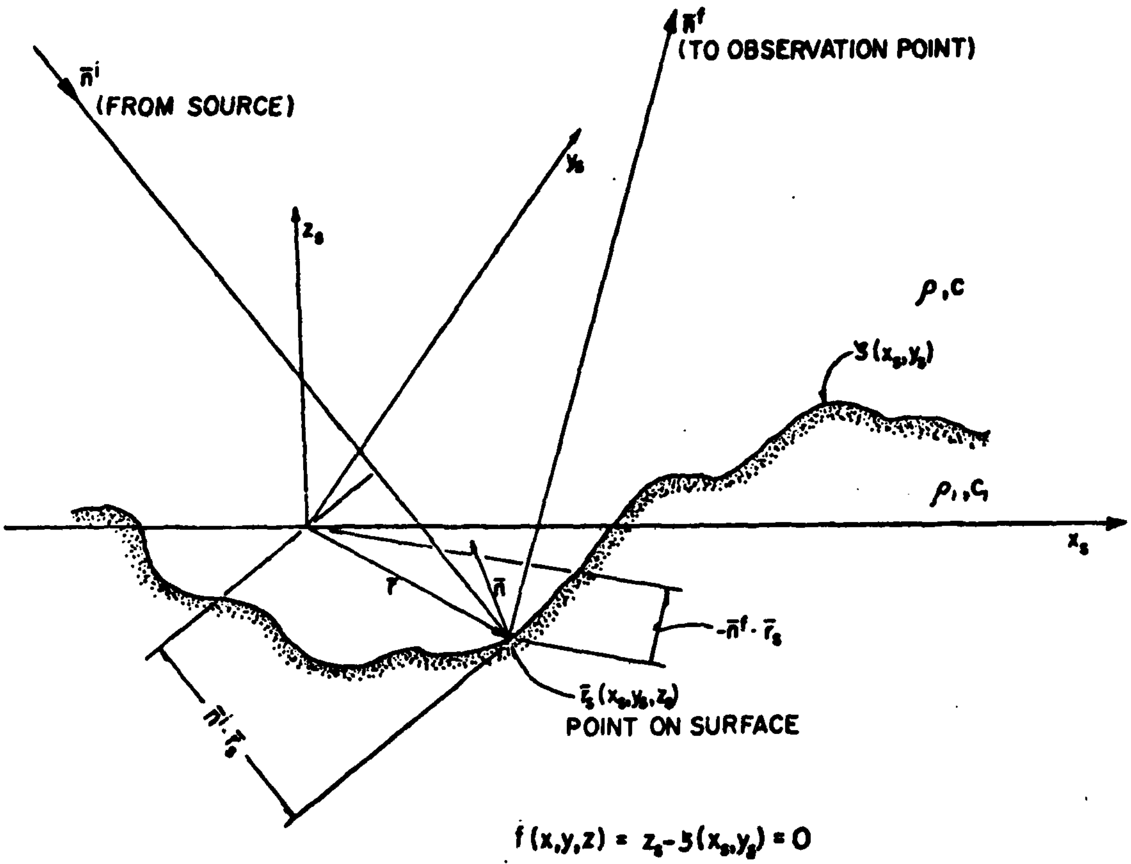



Figure 2.2   The scattered field from a rough surface.

For a uniform plane wave excitation assumed by Beckmann and Spizzichino (1963) $g(\bar{n}^i)$ is assumed to be constant and

$$E_o^i = 1 \tag{2.24}$$

The unit vector $\bar{n}$ normal to S is

$$\bar{n} = \nabla f/|\nabla f| = (-\frac{\partial\zeta}{\partial x}\bar{a}_x - \frac{\partial\zeta}{\partial y}\bar{a}_y + \bar{a}_z)/[(\frac{\partial\zeta}{\partial x})^2 + (\frac{\partial\zeta}{\partial y})^2 + 1]^{\frac{1}{2}} \tag{2.25}$$

and

$$dA_o = dx_s dy_s/\bar{n}\cdot\bar{a}_z = dx_s dy_s|\nabla f| \tag{2.26}$$

Thus subject to all the assumptions made by Beckmann the scattered field (2.18) reduces to (Beckmann and Spizzichino,1963; Ishimaru, 1978)

$$\psi^s(\bar{r}) = \frac{ie^{ikr_o^f}}{4\pi r_o^f}\int_S \bar{M}\cdot\nabla f e^{i\bar{v}\cdot\bar{r}_s}\, dx_s dy_s \tag{2.27}$$

where the integration is over the open surface S which has a non-vanishing reflection coefficient. For a flat surface normal to the unit vector

$$\bar{n}_o = n_{ox}\bar{a}_x + n_{oy}\bar{a}_y + n_{oz}\bar{a}_z \tag{2.28a}$$

$$\bar{n}_o\cdot\bar{r}_s = 0\ ,\ f(z_s) = \frac{\bar{n}_o\cdot r_s}{n_{oz}} = z_s + \frac{n_{ox}x_s + n_{oy}y_s}{n_{oz}} \tag{2.28b}$$

the scattered wave is given by

$$\psi^s_{\zeta=o}(\bar{r}) = \frac{ie^{ikr_o^f}}{4\pi r_o^f}\bar{M}\cdot\nabla f 4XY\,\mathrm{sinc}[(v_x - v_z\, n_{ox}/n_{oz})X]\mathrm{sinc}[(v_y - v_z\, n_{oy}/n_{oz})Y] \tag{2.29}$$

in which $\mathrm{sinc}\alpha = (\sin\alpha)/\alpha$, and

$$\bar{M}\cdot\nabla f = (R\bar{v} - \bar{p})\cdot\frac{\bar{n}_o}{n_{oz}} \tag{2.30}$$

For rough surfaces Beckmann obtains on assuming R = const. and on integrating by parts

$$\psi^s(\bar{r}) = \frac{ie^{ikr_o}}{4\pi r_o^f}\left[\left(\frac{\bar{M}\cdot\bar{v}}{v_z}\right)\int_S e^{i\bar{v}\cdot\bar{r}_s}\,dx_s dy_s + i\frac{M_y}{v_z}\int\left[e^{i\bar{v}\cdot\bar{r}_s}\right]_{-Y}^{Y}dx + i\frac{M_x}{v_z}\int\left[e^{i\bar{v}\cdot\bar{r}_s}\right]_{-X}^{X}dy\right] \tag{2.31}$$

Using the full wave approach (Bahar,1981a), it can be shown that for high frequencies R in (2.31) must be evaluated at the specular point where $\bar{n} = \bar{v}/v$.

The last two terms in (2.31) which do not depend on the shape of the surface are identified by Beckmann as an "edge effect." These terms are considered to be very small compared to the first term in (2.31) and therefore negligible (Beckmann and Spizzichino,1963). Since

$$\frac{\bar{M}\cdot\bar{v}}{v_z} = \frac{Rv^2}{v_z} = \frac{R(v_x^2 + v_y^2 + v_z^2)}{v_z} \tag{2.32}$$

the scattered field without the "edge effect" is given by

$$\psi^s(\bar{r}) = \frac{ie^{ikr_o^f}}{4\pi r_o^f}\,\frac{Rv^2}{v_z}\int_S e^{i\bar{v}\cdot\bar{r}_s}\,dx_s dy_s \tag{2.33}$$

On applying (2.33) to flat surfaces (2.28) one gets

$$\psi^s_{\zeta=o}(\bar{r}) = \frac{ie^{ikr_o^f}}{4\pi r_o^f}\left(\frac{Rv^2}{v_z}\right) 4XY\mathrm{sinc}[(v_x - v_z n_{ox}/n_{oz})X]\mathrm{sinc}[(v_y - v_z n_{oy}/n_{oz})Y] \tag{2.34}$$

The results for scattering by flat surfaces of area A (2.29)(which contains the "edge effect") and (2.34) (which does not contain the "edge effect") are now compared in detail. For scattering in the specular direction with respect to the plane $\bar{n}_o\cdot\bar{r}_s = 0$ (2.28).

$$\bar{n}_o = \bar{n}_{os} = \frac{\bar{n}^f - \bar{n}^i}{|\bar{n}^f - \bar{n}^i|} = -\,\bar{v}/v\ ,\ v = |\bar{v}| \tag{2.35a}$$

$$\bar{M}\cdot\nabla f = -(R\bar{v}-\bar{p})\cdot\bar{v}/vn_{oz} = -Rv^2/vn_{oz} = Rv^2/v_z \tag{2.35b}$$

Thus for the specular case the "edge effect" vanishes and the two solutions (2.29) and (2.34) are identical. The specularly scattered field from the flat surface $\bar{n}_o\cdot\bar{r}_s = 0$ (2.28) is

$$\psi^s_{spec}(\bar{r}) = \frac{ike^{ikr_o^f}}{2\, r_o^f} R \cos\theta_s A \tag{2.36}$$

in which R is the reflection coefficient for a wave incident at the angle $\theta_s$ upon the surface $\bar{n}_o \cdot \bar{r}_s = 0$. Since 4 XY is the projection of the surface $\bar{n}_o \cdot \bar{r}_s = 0$ on the xy plane, the area of this surface is

$$A = \frac{4\ XY}{\bar{n}\cdot\bar{a}_z} = \frac{4\ XY}{n_{oz}} \tag{2.37}$$

and

$$v \to 2k \cos\theta_s \quad , \quad v_z \to vn_z \tag{2.38}$$

in which $\theta_s$ is the angle of incidence and scatter for a specularly oriented surface, $\bar{n}_o = \bar{n}_{os}$, (2.35a). Thus, the Physical Optics solutions (2.29) and (2.34) for specular scattering by a plane of arbitrary slope satisfy the realizability relationship in electromagnetic theory. In the general case however, they could differ very significantly. To test these solutions for reciprocity interchange the location of transmitter and receiver. This amounts to the following substitutions in (2.27) and (2.33)

$$\bar{n}^i \to -\bar{n}^f \ , \ \bar{n}^f \to -\bar{n}^i \ , \ \bar{v} \to \bar{v} \ , \ \bar{p} \to -\bar{p} \tag{2.39}$$

Clearly therefore (2.33) satisfies reciprocity while (2.27) does not. Furthermore on setting $R \to 0$ solution (2.33) vanishes, as must be the case for a non-reflecting surface. However, on setting $R \to 0$ the solution (2.27) which includes the so-called "edge effect" reduces to

$$\psi^s(\bar{r}) = \frac{-ie^{ikr_o^f}}{4\pi r_o^f} \int_S \bar{p}\cdot\nabla f\ e^{i\bar{v}\cdot\bar{r}_s}\, dx_s dy_s \tag{2.40}$$

On integrating over the surface $\bar{n}_o \cdot \bar{r}_s = 0$ (2.40) becomes

$$\psi^s_{\zeta=o}(\bar{r}) = \frac{-ie^{ikr_o^f}}{4\pi r_o^f} (\bar{n}^i+\bar{n}^f)\cdot\bar{n}_o A\ \text{sinc}[(v_x-v_z n_{ox}/n_{oz})X]\text{sinc}[(v_y-v_z n_{oy}/n_{oz})Y] \tag{2.41}$$

The above expression vanishes only for backscatter ($\bar{n}^f = -\bar{n}^i$) or the specular case in which $\bar{n}_o = \bar{n}_{os}$ (2.35a) and

$$-(\bar{n}^i + \bar{n}^f)\cdot\bar{n}_{os} = \frac{\bar{p}\cdot\bar{v}}{v} = 0 \qquad (2.42)$$

Thus while the "edge effect" is not neglected in (2.27), this Physical Optics solution for the scattered field does not satisfy reciprocity nor does it vanish for the non-specular case as the reflection coefficient becomes vanishingly small. On the other hand the Physical Optics solution (2.33) without the term associated with the "edge effect" satisfies the reciprocity relationship (2.39) and vanishes as $R \to 0$. It also satisfies the realizability relationship (2.36). However, it has been shown that in general the so-called "edge term" is not necessarily very small in comparison to the first term in (2.31) which corresponds to the solution (2.33) (Fung, Moore and Parkins,1965). In these cases it has been suggested that "the 'edge effect' . . . may become important when considering scattering in directions far off specular". Yet it has been shown here that on retaining the "edge effect", the Physical Optics solution does not satisfy reciprocity and results in a peculiar non-vanishing scattered field for the non-specular direction (2.41).

The questions that need to be resolved therefore, are whether the "edge effect" should be suppressed only when it can be demonstrated that it is relatively small compared to the scattered field and if it needs to be included in the solution when it is not relatively small, how does one account for the fact that the resulting solution (2.27) does not satisfy reciprocity and does not vanish as the reflection coefficient becomes vanishingly small?

## 2.5 Evaluation of the Closed Surface Integral for the Scattered Field

In order to resolve the questions raised in Section 2.4, we examine the suggestion that "one may regard the rough surface S as part of any 'closed surface S' of which only S has a non-vanishing reflection coefficient, the remaining integral over S'-S is then easily shown to vanish so that there is no need to introduce the surface S'". To this end, the starting point of our present analysis is the "closed surface" integral (2.18) in which the vectors $\bar{M}, \bar{v}$ and $\bar{p}$ as well as the distances $r_s^i$ and $r_s^f$ are functions of position on the surface $A_o$ (spherical waves).

Divide the closed surface $A_o$ into two parts $S_1$ and $S_2$ and let the equation for the surfaces $S_1$ and $S_2$ be (see Fig. 2.1)

$$f_1(v,y,z) = z - \zeta_1(x,y) = 0 \tag{2.43}$$

and

$$f_2(x,y,z) = z - \zeta_2(x,y) = 0 \tag{2.44}$$

respectively, such that on $S_1$, $\bar{n}\cdot\bar{a}_z \geq 0$ and on $S_2$, $\bar{n}\cdot\bar{a}_z \leq 0$ and on the common closed line boundary C, $\bar{n}\cdot\bar{a}_z = 0$.

Thus

$$\bar{n}dA_o = dx_s dy_s \begin{cases} \nabla f_1 & \bar{n}\cdot\bar{a}_z \geq 0 \\ -\nabla f_2 & \bar{n}\cdot\bar{a}_z \leq 0 \end{cases} \tag{2.45}$$

where $\bar{n}$ is the inward normal to the closed surface $A_o$ and

$$f_1(x,y,z) = f_2(x,y,z) \text{ on C } (\bar{n}\cdot\bar{a}_z = 0) \tag{2.46}$$

Assume that the reflection coefficient R does not vary rapidly over the surface $A_o$ (i.e., ignore scattering due to changes in the reflection coefficient) and assume that the surface $A_o$ is not excited by the source at grazing angles such that

$$|g(\bar{n}^{-1})v_z| < K \text{ , K positive real const.} \tag{2.47}$$

Thus retaining only the far field terms it can be shown that

$$\frac{i}{4\pi}\nabla_s\cdot(\bar{M}g(\bar{n}^{-i})\exp ik(r_s^i + r_s^f)/r_s^i r_s^f v_z) \equiv \nabla_s\cdot\bar{F}$$

$$= i\,\bar{F}\cdot(\bar{v} - v_z\nabla f) \tag{2.48}$$

in which

$$r_s^i = [x_s^2 + y_s^2 + \zeta^2(x_s,y_s)]^{\frac{1}{2}} \tag{2.49a}$$

and

$$r_s^f = [(x_s - x)^2 + (y_s - y)^2 + (\zeta - z)^2]^{\frac{1}{2}} \tag{2.49b}$$

and f and $\zeta$ are defined in (2.43). Denoting the value of $\bar{F}$ (2.48) on $S_1$ and $S_2$ by $\bar{F}_1$ and $\bar{F}_2$ respectively and using the divergence theorem in two dimensions, it follows that since $\bar{F}_1 = \bar{F}_2$ on C

$$\int\nabla\cdot(\bar{F}_1 - \bar{F}_2)dx_s dy_s = \oint(\bar{F}_1 - \bar{F}_2)\cdot(\bar{a}_x dy_s - \bar{a}_y dx_s) = 0$$

$$= i\int[\bar{F}_1\cdot(\bar{v} - v_z\nabla f)_1 - \bar{F}_2\cdot(\bar{v} - v_z\nabla f)_2]dx_s dy_s \tag{2.50}$$

In view of (2.45), (2.50) reduces to

$$\int[\bar{F}_1\cdot v_{z1}\nabla f_1 - \bar{F}_2\cdot v_{z2}\nabla f_2]dx_s dy_s \equiv \oint v_z\bar{F}\cdot\bar{n}dA_o = \psi^s(\bar{r})$$

$$= \int[(\bar{F}\cdot\bar{v})_1 - (\bar{F}\cdot\bar{v})_2]dx_s dy_s \tag{2.51}$$

and since $\bar{M}\cdot\bar{v} = (R\bar{v} - \bar{p})\cdot\bar{v} = Rv^2$, (2.18) reduces to

$$\psi^s(\bar{r}) = \int\left[\frac{iRv^2}{4\pi v_z}g(\bar{n}^{-i})\exp\frac{[ik(r_s^i + r_s^f)]}{r_s^i r_s^f}\right]_2^1 dx_s dy_s \tag{2.52}$$

Thus the above expression does not contain the term associated with the "edge effect." The integrand in (2.52) corresponds to the specularly reflected wave from an elementary surface of area

$$dA_o = dx_s dy_s/\bar{n}_o\cdot\bar{a}_z \tag{2.53}$$

Thus the integrand of (2.52) is the contribution to the scattered field from an elementary surface and may be regarded as proportional to the "specific reflectance." In (2.52) instead of integrating over the closed surface $A_o$ one may integrate only over the surface S for which $R \neq 0$. Furthermore, if the distances from the source and observation points to points on the surface are very large compared to distances between any two points on the surface S, $\bar{n}^i$ and $\bar{n}^f$ can be assumed to be constant

(see Section 2.4) and (2.52) reduces to

$$\psi^s(\bar{r}) = \frac{iRv^2}{4\pi v_z} g(\bar{n}^i)\exp\frac{[ik(r_o^i + r_o^f)]}{r_o^i r_o^f}\int_S \exp(i\bar{v}\cdot\bar{r}_s)dx_s dy_s \quad (2.54)$$

Thus the terms identified with the "edge effect" (2.31) should be suppressed even when they are not relatively small. (Beckmann and Spizzichino,1963).

It should be noted that the above expression was derived under the assumption that $|g(\bar{n}^i)/v_z| < K$ (2.47). Thus (2.53) cannot be used for excitation at grazing angles. For grazing angles, the Physical Optics approach is not valid and a spectral approach must be used (Bahar,1981a).

To account for shadowing in the high frequency limit one should note that $\psi^s = -\psi^i$ on the shadow boundary since the total field $\psi = \psi^i + \psi^f$ vanishes. Furthermore, when the observation point is in the shadow region, the scattered field consists of the "reflected wave" $\psi^R$ and the "shadow-forming" wave (Morse and Feshbach,1953).

## 2.6 Concluding Remarks

It is shown here that the discrepancies between the different formulations for the scattered field and the problems associated with the so-called "edge effect" in the Physical Optics solution are due to the premature truncation of the closed surface integral expression for the scattered fields. On applying the divergence theorem (in two dimensions) to the closed surface integral (Section 2.5), the term identified as the "edge effect" is shown to vanish identically for all scatter directions. Thus using the Physical Optics approach (which cannot account for wave diffraction by edges), the so-called "edge effect" term must be surpressed even when it is not very small compared to the scattered field.

As a result the final form for the Physical Optics solution (2.52) satisfies reciprocity and the realizability relationships and it can be used for arbitrary excitation. The integrand in (2.52) is identified with the "specific reflectance" (per unit area) and it vanishes as the reflection coefficient for the rough surface vanishes.

## 2.7 References

1. Bahar, E.,"Full Wave Solutions for the Depolarization of the Scattered Radiation Fields by Rough Surfaces of Arbitrary Slope," IEEE Transactions on Antennas and Propagation, AP-29(3), pp. 443-454, 1981a.
2. Bahar, E., "Scattering and Depolarization by Rough Surfaces Near Grazing Angles—Full Wave Solution," to be published in IEEE Transactions on Antennas and Propagation, 1981b.
3. Barrick, D. E., "A More Exact Theory of Backscattering from Statistically Rough Surfaces," Ph.D. Thesis, Ohio State University, Columbus, Ohio. Also published as NASA Research Report Grant No. NsG-213-61, 1388-18, 31 August 1965.
4. Beckmann, P. and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, (Ch. 3), MacMillan, New York, 1963.
5. Boyd, M. L. and R. L. Deavenport, "Forward and Specular Scattering from a Rough Surface Theory and Experiment," Journal of the Acoustic Society of America, 53, pp. 791-801, 1973.
6. Brekhovskikh, L. M., Waves in Layered Media, (Ch. 1), Academic Press, New York, 1960.
7. Eckart, G., "The Scattering of Sound from the Sea Surface," Journal of the Acoustic Society of America, 25, pp. 566-570, 1953.

8. Fung, A.K., R.K. Moore and B.E. Parkins, "Notes on Backscattering and Depolarization by Gently Undulating Surfaces," Journal of Geophysical Research, 70(6), 1965.

9. Ishimaru, A., Wave Propagation and Scattering in Random Media, Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing, (Vol. 2, Ch. 21), Academic Press, 1978.

10. Jordan, E. G., and K. G. Balmain, Electromagnetic Waves and Radiating Systems, (Ch. 10), Prentice Hall, Inc., New Jersey, 1968.

11. Morse and Feshbach, Methods of Theoretical Physics, (Ch. 11), McGraw-Hill, New York, 1953.

12. Parkins, B. E., "Scattering from the Time-Varying Surface of the Ocean," Journal of the Acoustic Society of America, 42, pp. 1262-1267, 1967.

13. Valenzuela, G. R., "Scattering of Electromagnetic Waves from an Isotropic Gaussian Surface," The Johns Hopkins University Applied Physics Laboratory Report BPD66V-2.

14. Wetzel, L., "H.F. Sea Scatter and Ocean Wave Spectra," Abstract in Proceedings of URSI Spring Meeting April 18-21, 1966. (Also in Institute for Defense Analysis Report), 1965.

15. Wetzel, L., "Low Frequency Acoustic Scatter from the Sea Surface--Theory," Naval Research Laboratory Report prepared for the International Workshop on Low Frequency Propagation and Noise, Monterey, CA., 12-14 November 1975.

## 2.A Appendix

For the convenience of the reader, the formulations for the scattered fields derived by Eckart (1953), Parkins (1967) and Boyd-Deavenport (1973) are summarized here for the Direchlet condition and point source excitation $(s(\bar{r}) = \delta(\bar{r})$ (2.1)). Thus on substituting

$$\psi(\bar{r}_o^s) = \psi^i(\bar{r}_o^s) + \psi^s(\bar{r}_o^s) = 0, \; \psi^i(\bar{r}_o^s) = \frac{e^{ikr_s^i}}{r_s^i}, \; (R \to -1) \tag{2A.1}$$

and the Kirchhoff approximation

$$\left[\frac{\partial\psi}{\partial n} = \frac{\partial\psi^i}{\partial n} + \frac{\partial\psi^s}{\partial n} \approx 2\,\frac{\partial\psi^i}{\partial n}\right]_{\bar{r}_o^s} \approx 2ik\;\bar{n}^i\cdot\bar{n}\;\psi^i(\bar{r}_o^s) \tag{2A.2}$$

into (2.10), and on replacing the closed surface $A_o$ by the open surface S one obtains Parkins formulation

$$\psi^s(\bar{r}) = \frac{-ik}{2\pi}\int \bar{n}^i\cdot\bar{n}\;\psi^i(\bar{r}_o^s)\,\frac{e^{ikr_s^f}}{r_s^f}\,dS \tag{2A.3}$$

If on the other hand (2A.1) and (2A.2) are substituted into (2.16a), one obtains Eckart's formulation

$$\psi^s(\bar{r}) = \frac{-ik}{4\pi}\int (\bar{n}^i-\bar{n}^f)\cdot\bar{n}\;\psi^i(\bar{r}_o^s)\,\frac{e^{ikr_s^f}}{r_s^f}\,dS \tag{2A.4}$$

Boyd and Deavenport assume that the Green's function $G(\bar{r}|\bar{r}_o)$ satisfies (2.7) as well as the boundary conditions (2A.1) and (2A.2) at S. Thus using (2.16a) as the starting point and substituting $G(\bar{r}|\bar{r}_o^s) = 0$, $\partial G(\bar{r}|\bar{r}_o^s)/\partial n \approx -2ik\;\bar{n}^f\cdot\bar{n}\;G(\bar{r}|\bar{r}_o^s)$ one obtains the Boyd-Deavenport formulation:

$$\psi^s(\bar{r}) = \frac{ik}{2\pi}\int \bar{n}^f\cdot\bar{n}\;\psi^i(\bar{r}_o^s)\,\frac{e^{ikr_s^f}}{r_s^f}\,dS \tag{2A.5}$$

The three formulations (2A.3), (2A.4) and (2A.5) will obviously give different results except for backscatter $\bar{n}^f = -\bar{n}^i$. Note also that in all

the above formulations the integrands are equal at the specular (stationary phase) points where $\bar{n} = \bar{n}_{os}$ (2.35a) and

$$\frac{(\bar{n}^f - \bar{n}^i)\cdot\bar{n}_{os}}{2} = -\bar{n}^i\cdot\bar{n}_{os} = \bar{n}^f\cdot\bar{n}_{os} = \frac{1 - \bar{n}^f\cdot\bar{n}^i}{|\bar{n}^f - \bar{n}^i|} \qquad (2A.6)$$

Thus the above three physical optics formulations (2A.3), (2A.4) and (2A.5) can be shown to be in agreement, provided that the respective integrals are evaluated using the stationary phase method (Bahar,1981a). However, the discrepancies between the above three formulations increases as non-specular scattering becomes more important. When the most significant contributions to the scattered fields do not come from specular points (as in the case of backscatter near grazing incidence) the Physical Optics or Kirchhoff approximations are not valid and other methods must be used to determine the scattered fields (Bahar,1981a).

In Section 2.5, it is shown that either (2.10) or (2.16a) can be used for the starting point of the analysis provided that integration is over the closed surface $A_o$. Thus on following the analytical procedures of Section 2.4 and noting that

$$k\bar{n}^i\cdot\bar{v} = -k\bar{n}^f\cdot\bar{v} = \frac{k(\bar{n}^i - \bar{n}^f)\cdot\bar{v}}{2} = \frac{v^2}{2} = k^2(1-\bar{n}^f\cdot\bar{n}^i) \qquad (2A.7)$$

all the above three formulations can be shown to reduce to (2.5) with $R = -1$ and $g(\bar{n}^i) = 1$ provided that in (2A.3), (2A.4) and (2A.5), the integration is over the closed surface $A_o$.

## 3.0 DEPOLARIZATION OF THE SCATTERED RADIATION FIELDS BY CONDUCTING OBJECTS OF IRREGULAR SHAPE ABOVE ROUGH LAND AND SEA—FULL WAVE SOLUTIONS

### 3.1 Background

In this section the full wave approach to problems of scattering by rough surfaces has been applied to the problem of depolarization of the scattered radiation fields by objects of finite conductivity and irregular shape. In the analysis complete expansions are employed, exact boundary conditions are imposed and a variable coordinate system that conforms with the local features of the irregular surface is used. The full wave solutions are expressed in forms that can be readily compared with earlier solutions and they can be used to reconcile the differences and bridge the wide gap between these solutions. Thus, the full wave solutions for the backscatter cross-section are shown to reduce to the Physical Optics solutions when the high frequency, stationary phase approximations are used. Similarly, for slightly rough surfaces the full wave expressions reduce to the perturbational solutions for the backscatter cross-section.

The full wave solutions are shown to be consistent with the duality reciprocity and realizability relationships in electromagnetic theory. These solutions are invariant to coordinate transformations. Since upward and downward scattering are considered in the analysis, multiple scattering and shadowing effects can be taken into account in a self-consistent manner. Thus, the total scattered field, varies continuously as the observer moves across a shadow boundary and there is no need to

introduce transition terms derived from other theories. The full wave approach can be applied to deterministic, periodic rough surfaces. It can also be used to determine the scattering by finite scatterers in the presence of rough land or sea.

Scattering by random rough surfaces is dealt with in detail in Section 3.5. The deterministic scattering problem considered here in detail provides the basis for such an analysis.

## 3.2 Discussion

In this paper, the full wave approach has been applied to the problem of depolarization of the scattered radiation fields by objects of finite conductivity and irregular shape. Since the full wave approach (Bahar,1981a) can account for multiple scattering, the irregularly shaped objects may be in the vicinity of rough land and sea. Thus these solutions could also be used to distinguish between the radar returns from the irregularly shaped object and the clutter from rough surfaces in the background.

The principal elements of the full wave approach are as follows: (Bahar,1981a) Complete expansions of the electromagnetic fields are employed. Thus, the electric and magnetic fields are expressed in terms of the radiation term as well as the lateral wave and the surface wave terms. Since in general the irregularly shaped objects and rough surfaces depolarize the incident waves, the complete expansions include both vertically and horizontally polarized waves (Bahar,1973a). Exact boundary conditions for the total fields are imposed at the irregularly shaped surfaces. Furthermore, the medium is characterized by the electromagnetic parameters, $\varepsilon$ and $\mu$ (which may vary along the propagation path) and it

is not necessary to employ approximate impedance boundary conditions (Bahar and Rajan, 1979). Precise mathematical procedures are used in the analysis. Since the characteristic functions used in the complete field expansions do not individually satisfy the boundary conditions at the irregular surfaces, in general the field expansions do not converge uniformly on the boundaries. Thus, for example in order to avoid term-by-term differentiation of the field expansions, use is made of Green's theorems (Bahar, 1973b). Maxwell's equations for the transverse components of the electric and magnetic fields are converted into a rigorous set of generalized telegraphists' equations for the forward and backward traveling wave amplitudes. These first order, ordinary, coupled differential equations can be solved using numerical or analytical techniques (Bahar, 1973b). The solutions for the coupled wave amplitudes are substituted into the complete field expansions and the steepest descent method is used to obtain the expressions for the scattered radiation fields (Bahar, 1981a). A variable coordinate system that conforms with the local features of the irregular boundary is used to remove the restrictions on the gradient of the irregular boundary (Bahar, 1980). (See Appendix 3.A)

The principal advantages of the full wave approach are as follows: The full wave solutions are valid for all incident and scatter angles including grazing angles, Brewster angles, scattering in the specular direction and backscatter. Since upward and downward scattering are considered in the full wave analysis, multiple scattering and shadowing effects can be taken into account in a self-consistent manner. Moreover, the total scattered fields vary continuously as the observer moves across a shadow boundary and there is no need to introduce transition terms

derived from other theories. Since exact boundary conditions are imposed in the analysis and there are no restrictions on the gradient of the rough surface, it is not necessary to characterize the rough surface by an approximate impedance boundary. Furthermore, since precise mathematical procedures are followed in the derivation of the generalized telegraphists' equations, the full wave approach can be used even when the characteristic functions, used in the complete expansions, do not individually satisfy the boundary conditions at the irregular surfaces. The full wave solutions are shown to be consistent with duality, reciprocity and realizability relations in electromagnetic theory. They account for coupling between the radiation fields, the lateral waves and the surface wave terms of the complete field expansions (Bahar, 1977). The full wave solutions are valid from low frequencies up to optical frequencies provided that the illuminated surface is at least several wavelengths wide. They can be expressed in a form that can be readily compared with earlier solutions that have limited applications. Thus these solutions can be used by the engineer who is not necessarily familiar with the analytical techniques used in the derivations. Furthermore, the full wave approach can be used to resolve the discrepancies that exist between earlier solutions. Thus the perturbations solutions (Rice, 1951; Barrick, 1970), can be reconciled with the corresponding Physical Optics solutions (Beckmann and Spizzichino, 1963; Beckmann, 1968). The full wave approach can be applied to deterministic, periodic and random rough surfaces. These solutions are invariant to coordinate transformations and they can be applied to problems of scattering by irregularly shaped objects of finite conductivity as well as to problems of scattering by rough land and sea.

In Section 3.3, the problem is formulated and the principal elements of the full wave approach are summarized. In Section 3.4, the full wave solutions are presented for the scattered radiation fields by irregular objects in the vicinity of rough land or sea. In this section the effects of shadowing and multiple scattering are accounted for. In Section 3.5 the invariance properties of the full wave solutions as well as the duality and reciprocity relationships are examined in detail. In Section 3.6 the stationary phase approach is used to obtain the high frequency approximations of the full wave solutions. These solutions are compared with earlier Physical Optics solutions that are based on the Kirchhoff approach. In Section 3.7 realizability is examined and energy conservation is shown to be satisfied. The full wave expressions for the backscatter cross-sections are derived and they are shown to be in agreement with the corresponding perturbational solutions if the gradient of the rough surface is assumed to be small. Thus, the discrepancies between the perturbational and the Physical Optics solutions are resolved and the wide gap that exists between them is bridged by the full wave solutions. In Section 3.8 the full wave solutions are applied to grazing, specular and Brewster angles. Thus, if at grazing angles, stationary phase points do not exist on the rough surface, the Physical Optics solutions fail even at very high frequencies. Special forms of the full wave solutions are given for good conducting boundaries.

## 3.3 Formulation of the Problem

To determine the reflection and transmission of electromagnetic waves at the interface between two isotropic media characterized by different permittivities, $\epsilon$, and permeabilities, $\mu$, it is necessary to decompose the incident (unperturbed) electromagnetic radiation (far)

fields into vertically and horizontally polarized waves. When the interface is a plane normal to the unit vector $\bar{a}_y$ for instance, the plane of incidence for a wave traveling in the direction $\bar{n}^i$ is defined as the plane normal to the unit vector $\bar{a}_{Hi}$, where

$$\bar{a}_{Hi} = (\bar{n}^i \times \bar{a}_y)/|\bar{n}^i \times \bar{a}_y| \tag{3.1}$$

Since the radiation fields are normal to $\bar{n}^i$, the incident electric fields can be expressed in terms of the orthogonal pairs of unit vectors $\bar{a}_{Hi}$ and $\bar{a}_{Vi}$ as follows:

$$\bar{E}^i = E^{Vi}\,\bar{a}_{Vi} + E^{Hi}\,\bar{a}_{Hi} \tag{3.2}$$

where

$$\bar{a}_{Vi} = \bar{a}_{Hi} \times \bar{n}^i \tag{3.3}$$

The corresponding magnetic field components for the vertically and horizontally polarized waves are

$$\begin{pmatrix} H^{Vi} \\ H^{Hi} \end{pmatrix} = \frac{1}{\eta}\begin{pmatrix} E^{Vi} \\ E^{Hi} \end{pmatrix} \equiv G^i/\eta \tag{3.4}$$

in which $\eta = (\mu/\varepsilon)^{1/2}$ is the intrinsic impedance. Thus

$$\bar{H}^i = (H^{Vi}\,\bar{a}_{Vi} + H^{Hi}\,\bar{a}_{Hi}) \times \bar{n}^i \tag{3.5}$$

When the boundary between the two media is irregular, as in the case of a rough surface $f(x,y,z) = 0$, it is convenient to decompose the radiation fields into vertically and horizontally polarized components $E_n^{Vi}$ and $E_n^{Hi}$ with respect to the "local tangent plane." This plane is perpendicular to the varying unit vector $\bar{n}$ which is the outward normal from the scattering object (see Fig. 3.1). Thus in this case the incident radiation fields are expressed as

$$\bar{E}^i = E_n^{Vi}\,\bar{a}_{Vi}^n + E_n^{Hi}\,\bar{a}_{Hi}^n \tag{3.6}$$

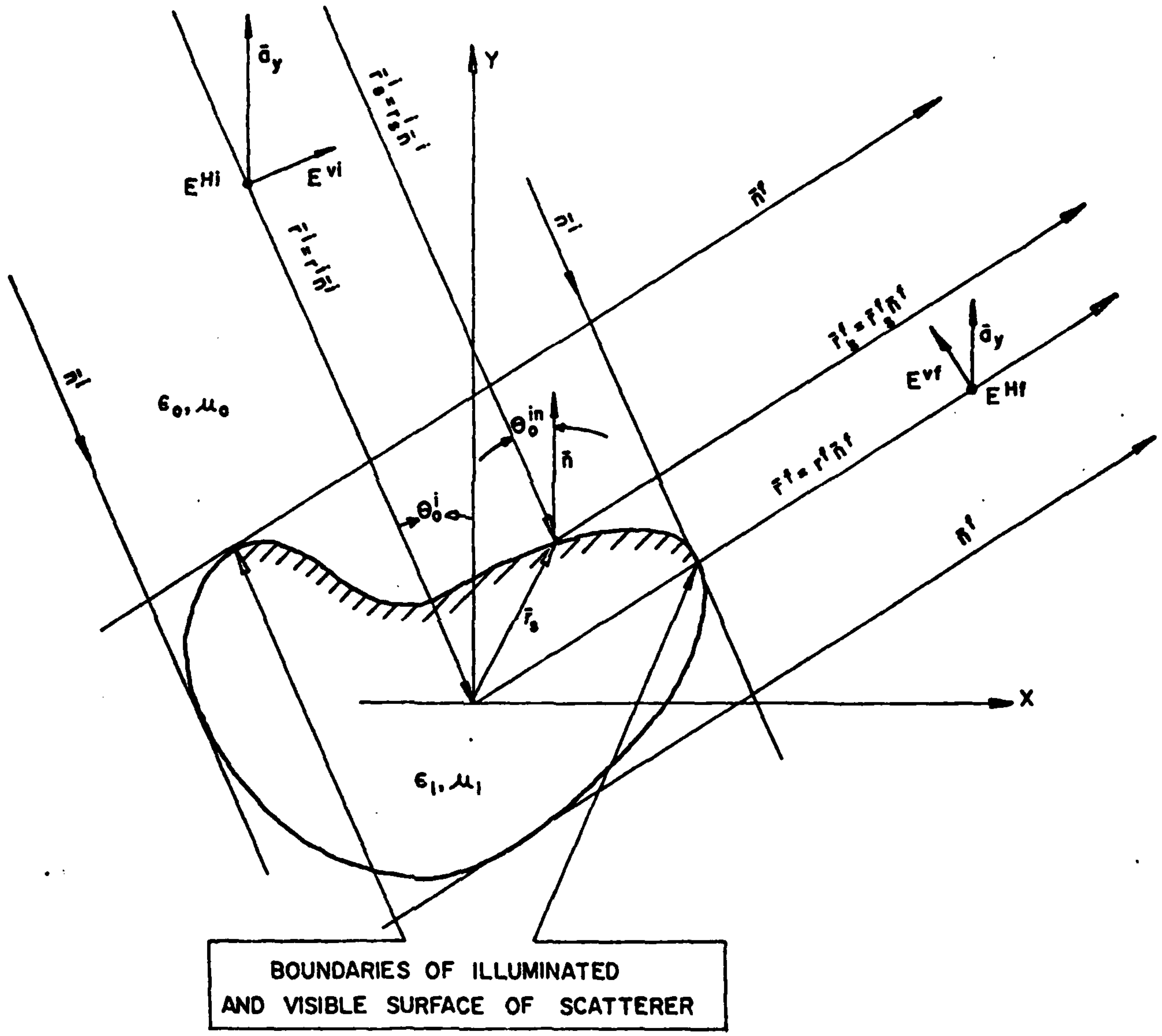



Figure 3.1 Depolarization of the scattered field from an irregularly shaped object.

where

$$\bar{a}^{n}_{Hi} = (\bar{n}^{i} \times \bar{n})/|\bar{n}^{i} \times \bar{n}| \tag{3.7}$$

The relationship between the respective components of the vertically and horizontally polarized waves is

$$G^{in} \equiv \begin{pmatrix} E^{Vi}_{n} \\ E^{Hi}_{n} \end{pmatrix} = \begin{pmatrix} C^{i}_{\psi} & S^{i}_{\psi} \\ -S^{i}_{\psi} & C^{i}_{\psi} \end{pmatrix} \begin{pmatrix} E^{Vi} \\ E^{Hi} \end{pmatrix} \equiv T^{i} G^{i} \quad , \tag{3.9}$$

where $G^{i}$ is defined in (3.4) and $C^{i}_{\psi}$ and $S^{i}_{\psi}$ are the cosine and sine of the angle between the planes of incidence normal to $\bar{a}_{Hi}$ and $\bar{a}^{n}_{Hi}$. Thus $C^{i}_{\psi}$ and $S^{i}_{\psi}$ are expressed in terms of the dot and scalar triple products,

$$C^{i}_{\psi} = \cos\psi^{i} = \bar{a}_{Hi} \cdot \bar{a}^{n}_{Hi} \text{ and } S^{i}_{\psi} = \sin\psi^{i} = \left[\bar{a}_{Hi} \bar{a}^{n}_{Hi} \; \bar{n}^{i}\right] \tag{3.10}$$

Since the matrix $T^{i}$ in (3.9) is Hermetian

$$(T^{i})^{T} = (T^{i})^{-1} \tag{3.11}$$

The invariance of the vector $\bar{E}$ in (3.2) and (3.7) requires that

$$\begin{pmatrix} \bar{a}^{n}_{Vi} \\ \bar{a}^{n}_{Hi} \end{pmatrix} = T^{i} \begin{pmatrix} \bar{a}_{Vi} \\ \bar{a}_{Hi} \end{pmatrix} \tag{3.12}$$

In a similar manner the radiation field $\bar{E}^{f}$, scattered in the direction $\bar{n}^{f}$ can be expressed in terms of vertically and horizontally polarized components with respect to the reference plane normal to $\bar{a}_{y}$ or with respect to the plane normal to $\bar{n}$. Thus

$$\bar{E}^{f} = E^{Vf} \bar{a}_{Vf} + E^{Hf} \bar{a}_{Hf} = E^{Vf}_{n} \bar{a}^{n}_{Vf} + E^{Hf}_{n} \bar{a}^{n}_{Hf} \tag{3.13}$$

The relationship between the respective components of the vertically and horizontally polarized waves is

$$G^{f} \equiv \begin{pmatrix} E^{Vf} \\ E^{Hf} \end{pmatrix} = \begin{pmatrix} C^{f}_{\psi} & -S^{f}_{\psi} \\ S^{f}_{\psi} & C^{f}_{\psi} \end{pmatrix} \begin{pmatrix} E^{Vf}_{n} \\ E^{Hf}_{n} \end{pmatrix} \equiv T^{f} G^{fn} \tag{3.14}$$

in which $C_\psi^f$ and $S_\psi^f$ are the cosines and sines of the angles between the planes of scatter normal to $\bar{a}_{Hf}$ and $\bar{a}_{Hf}^n$. The expressions for the unit vectors $\bar{a}_{Vf}$, $\bar{a}_{Hf}$, $\bar{a}_{Vf}^n$ and $\bar{a}_{Hf}^n$ as well as for $C_\psi^f$ and $S_\psi^f$ are obtained from the above expressions for $\bar{a}_{Vi}$, $\bar{a}_{Hi}$, $\bar{a}_{Vi}^n$, $\bar{a}_{Hi}^n$, $C_\psi^i$ and $S_\psi^i$ on replacing $\bar{n}^i$ with $\bar{n}^f$.

The full wave expression for the radiation field scattered by an elementary area $d\bar{A}$ normal to the unit vector $\bar{n}$ can be expressed in matrix form as follows, (see Fig. 3.1, Bahar, 1981a).

$$dG^f = G_o C_o^{in} T^f F(\bar{n}^{fn}, \bar{n}^{in}) T^i \exp[ik_o(\bar{n}^f - \bar{n}^i)\cdot\bar{r}_s] d\bar{A}\cdot\bar{n}\, G^i$$
$$\equiv G_o\, dC(\bar{n}^f, \bar{n}^i) G^i \tag{3.15}$$

The constant $G_o$ is given by

$$G_o = -ik_o \exp[-ik_o r^f]/2\pi r^f \tag{3.16}$$

in which $k_o = \omega(\mu_o \varepsilon_o)^{1/2}$ is the free space wave number, $\mu_o$ and $\varepsilon_o$ are the free space permeability and permittivity respectively, and $r^f$ is the distance from the origin to the observation point. The assumed $\exp(i\omega t)$ time dependence is factored throughout this work. The transformation matrices $T^i$ and $T^f$ are defined in (3.10) and (3.14). The elementary area $d\bar{A}$ is given by

$$d\bar{A} = \bar{n}\, dx\, dz/(\bar{n}\cdot\bar{a}_y) \tag{3.17}$$

and the matrix $G^i$ for the incident fields (3.4) depends upon the excitation. Thus for electric and magnetic dipoles oriented in the $\bar{a}_y$ direction (Bahar and Rajan, 1979)

$$G^{Pi} = E^{Pi} = \eta_o H^{Pi} = \frac{ik_o \sin\theta_o^i \exp[-ik_o r^i]}{4\pi r^i} \begin{cases} \eta_o J, & P=V \\ -M, & P=H \end{cases} \tag{3.18}$$

in which $r^i$ is the distance from the dipoles to the origin and J (Amp-meters) and M (Volt-meters) are the electric and magnetic dipole moments. The angle $\theta_o^i$ is the angle of incidence in medium 0 with respect to the

reference plane normal to $\bar{a}_y$. Thus, (see Fig. 3.1)

$$\cos\theta_o^i \equiv C_o^i = -\bar{n}^i \cdot \bar{a}_y \tag{3.19}$$

The angle $\theta_o^{in}$ is the angle of incidence with respect to local tangent plane normal to $\bar{n}$. Thus (see Fig. 3.1 and Appendix 3.A)

$$\cos\theta_o^{in} = C_o^{in} = -\bar{n}^i \cdot \bar{n} \tag{3.20}$$

In (3.15), $\bar{r}_s$ is the position vector from the origin to the elementary area $d\bar{A}$ on the boundary of the scatterer. The surface of the scatterer is given by

$$f(x,y,z) = 0 = \begin{cases} y-h_1(x,z) & , \ \bar{n}\cdot\bar{a}_y \geq 0 \\ y-h_2(x,z) & , \ \bar{n}\cdot\bar{a}_y \leq 0 \end{cases} \tag{3.21a}$$

Thus

$$\bar{r}_s = x\ \bar{a}_x + h(x,z)\bar{a}_y + z\ \bar{a}_z = \bar{r} \mp f(x,y,z)\bar{a}_y \tag{3.21b}$$

The unit vector $\bar{n}$ is the outward normal from the scatterer. Thus

$$\bar{n} = \pm \nabla f/|\nabla f| \equiv \sin\gamma\ \cos\delta\ \bar{a}_x + \cos\gamma\ \bar{a}_y + \sin\gamma\ \sin\delta\ \bar{a}_z$$

$$= \pm\ [-\ h_x\ \bar{a}_x + \bar{a}_y - h_z\ \bar{a}_z]/(h_x^2 + 1 + h_z^2)^{\frac{1}{2}} \tag{3.22a}$$

in which the upper and lower signs are used for surfaces $y=h_1$ or $y=h_2$ respectively and

$$h_x = \partial h/\partial x \quad \text{and} \quad h_z = \partial h/\partial z \tag{3.22b}$$

The scatterer is characterized by the complex permittivity and permeability $\epsilon_1$ and $\mu_1$ respectively. For simplicity in this paper the scatterer is assumed to be opaque, therefore transmission through the scattering object is neglected. The unit vectors $\bar{n}^i$ and $\bar{n}^f$ in the directions of the incident and scattered waves respectively, can be expressed as follows in terms of the reference coordinate system

$$\bar{n}^i = \sin\theta_o^i\ \cos\phi^i\ \bar{a}_x - \cos\theta_o^i\ \bar{a}_y + \sin\theta_o^i\ \sin\phi^i\ \bar{a}_z \tag{3.23}$$

and

$$\bar{n}^f = \sin\theta_o^f \cos\phi^f \,\bar{a}_x + \cos\theta_o^f \,\bar{a}_y + \sin\theta_o^f \sin\phi^f \,\bar{a}_z \tag{3.24}$$

The elements $F^{PQ}$ of the 2x2 scattering matrix $F(\bar{n}^{fn},\bar{n}^{in})$, (3.15) are given by (Bahar, 1981a).

$$C_o^{in}F^{VV} = \frac{2C_o^{in}\,C_o^{fn}[(\mu_r C_1^{in}C_1^{fn}\,\cos(\phi^{fn}-\phi^{in})-S_o^{in}S_o^{fn})(1-1/\varepsilon_r)+(1-\mu_r)\cos(\phi^{fn}-\phi^{in})]}{(C_o^{in}+\eta_r C_1^{in})(C_o^{fn}+\eta_r C_1^{fn})(C_o^{in}+C_o^{fn})} \tag{3.25a}$$

$$C_o^{in}F^{HH} = \frac{2C_o^{in}C_o^{fn}[(\varepsilon_r C_1^{in}C_1^{fn}\,\cos(\phi^{fn}-\phi^{in})-S_o^{in}S_o^{fn})(1-1/\mu_r)+(1-\varepsilon_r)\cos(\phi^{fn}-\phi^{in})]}{(C_o^{in}+C_1^{in}/\eta_r)(C_o^{fn}+C_1^{fn}/\eta_r)(C_o^{in}+C_o^{fn})} \tag{3.25b}$$

$$C_o^{in}F^{HV} = -\frac{\sin(\phi^{fn}-\phi^{in})2C_o^{in}C_o^{fn}n_r[(1-1/\varepsilon_r)C_1^{in}-(1-1/\mu_r)C_1^{fn}]}{(C_o^{in}+\eta_r C_1^{in})(C_o^{fn}+C_1^{fn}/\eta_r)(C_o^{in}+C_o^{fn})} \tag{3.25c}$$

$$C_o^{in}F^{VH} = -\frac{\sin(\phi^{fn}-\phi^{in})2C_o^{in}C_o^{fn}n_r[(1-1/\mu_r)C_1^{in}-(1-1/\varepsilon_r)C_1^{fn}]}{(C_o^{in}+C_1^{in}/\eta_r)(C_o^{fn}+\eta_r C_1^{fn})(C_o^{in}+C_o^{fn})} \tag{3.25d}$$

In which the dimensionless quantities $n_r$, $\eta_r$, $\varepsilon_r$, and $\mu_r$ are $n_r = (\varepsilon_1\mu_1/\varepsilon_o\mu_o)^{\frac{1}{2}}$, $\eta_r = \eta_1/\eta_o$, $\varepsilon_r = \varepsilon_1/\varepsilon_o$ and $\mu_r = \mu_1/\mu_o$.

The sines of the angles $\theta_1^{in}$ and $\theta_1^{fn}$ for medium 1 are given by Snell's law

$$S_1^{in} = \sin\theta_1^{in} = S_o^{in}/n_r,\quad S_1^{fn} = \sin\theta_1^{fn} = S_o^{fn}/n_r \tag{3.26}$$

Thus

$$C_1^{in} = \cos\theta_1^{in} = [1-(S_1^{in})^2]^{\frac{1}{2}},\quad C_1^{fn} = \cos\theta_1^{fn} = [1-(S_1^{fn})^2]^{\frac{1}{2}} \tag{3.27}$$

Furthermore, the angle between the local plane of incidence and the local scatter plane is given by

$$\cos(\phi^{fn}-\phi^{in}) = \bar{a}_{Hf}^{n}\cdot\bar{a}_{Hi} = \frac{C_o^{fn}C_o^{in}-C_o^f C_o^i+S_o^f S_o^i\,\cos(\phi^f-\phi^i)}{S_o^{fn}\,S_o^{in}} \tag{3.28}$$

and

$$\sin(\phi^{fn}-\phi^{in}) = [\bar{a}_{Hf}^{n}\;\bar{a}_{Hi}^{n}\,\bar{n}] = [\sin\gamma\left\{C_o^i S_o^f\sin(\phi^f-\delta)-C_o^f S_o^i\sin(\phi^i-\delta)\right\} + \cos\gamma\; S_o^f\, S_o^i\,\sin(\phi^f-\phi^i)]/S_o^{fn}S_o^{in} \tag{3.29}$$

## 3.4 Full Wave Solutions for the Scattered Radiation Fields--Effects of Shadowing and Multiple Scatter

The total scattered radiation fields are obtained by integrating the expression for the differential scattered field (3.15) over the surface of the scatterer which is both illuminated and visible at the observation point. All four elements of the scattering matrix $C_o^{in}$ F, in (3.25) vanish as

$$C_o^{in} = -\bar{n}^i \cdot \bar{n} = 0 \quad , \; \theta_o^{in} \to \pm \pi/2 \tag{3.30a}$$

or

$$C^{fn} = \bar{n}^f \cdot \bar{n} = 0 \quad , \; \theta_o^{fn} \to \pm \pi/2 \tag{3.30b}$$

Thus, the scattered radiation fields vanish in a continuous manner as the observer moves across the shadow boundary. In the shadow regions however, the total electromagnetic fields do not vanish since the complete expansions of the fields constitute the radiation term as well as the lateral and surface wave terms (Bahar, 1973a,b, 1977, 1978).

The nonilluminated (shadow) boundary is determined by points on the scatterer $\bar{r}_s = \bar{r}_{s1}^i$ that satisfy (see Fig. 3.1)

$$\bar{n}^i \cdot \bar{n}(\bar{r}_{s1}^i) = g^i(x_{s1}^i, z_{s1}^i) = 0 \tag{3.31a}$$

Thus substituting (3.22) and (3.23) into (3.31a), the boundary of the nonilluminated region is given by

$$\tan\theta_o^i [h_x^i \cos\phi^i + h_z^i \sin\phi^i] = -1 \tag{3.31b}$$

where

$$h_x^i = h_x(\bar{r}_{s1}^i) \text{ and } h_z^i = h_z(\bar{r}_{s1}^i) \tag{3.31c}$$

This nonilluminated region extends to points on the scatterer, $\bar{r}_s = \bar{r}_{s2}^i$ that satisfy

$$(\bar{r}_{s2}^i - \bar{r}_{s1}^i) \cdot \bar{n}(\bar{r}_{s1}^i) = 0 \tag{3.32a}$$

Thus

$$h_2^i - h_1^i = (x_{s2}^i - x_{s1}^i) h_x^i + (z_{s2}^i - z_{s1}^i) h_z^i \tag{3.32b}$$

where

$$h_1^i = h(\bar{r}_{s1}^i) \ , \ h_2^i = h(\bar{r}_{s2}^i) \tag{3.32c}$$

Similarly the region of the scatterer which is nonvisible to the observer extends from points on the scatterer where $\bar{r}_s = \bar{r}_{s1}^f$ to points where $\bar{r}_s = \bar{r}_{s2}^f$. The loci of $\bar{r}_s = \bar{r}_{s1}^f$ is given by

$$\bar{n}^f \cdot \bar{n}(\bar{r}_{s1}^f) = g^f(x_{s1}^f, z_{s1}^f) = 0 \tag{3.33a}$$

Thus

$$\tan\theta_o^f[h_x^f \cos\phi^f + h_z^f \sin\phi^f] = 1 \tag{3.33b}$$

where

$$h_x^f = h_x(\bar{r}_{s1}^f) \ , \ h_z^f = h_z^f(\bar{r}_{s1}^f) \tag{3.33c}$$

The loci of $\bar{r}_s = \bar{r}_{s2}^f$ is given by

$$(\bar{r}_{s2}^f - \bar{r}_{s1}^f)\cdot\bar{n}(\bar{r}_{s1}^f) = 0 \tag{3.34a}$$

Thus

$$h_2^f - h_1^f = (x_{s2}^f - x_{s1}^f)h_x^f + (z_{s2}^f - z_{s1}^f)h_z^f \tag{3.34b}$$

where

$$h_1^f = h(\bar{r}_{s1}^f) \ , \ h_2^f = h(\bar{r}_{s2}^f) \tag{3.34c}$$

Define the shadow function $D(\bar{r}_s)$ such that

$$D(\bar{r}_s) = \begin{cases} 1 \ , & \text{illuminated and visible region} \\ 0 \ , & \text{nonilluminated or nonvisible region} \end{cases} \tag{3.35}$$

where the nonilluminated region is defined by (3.31) and (3.32) while the nonvisible region is defined by (3.33) and (3.34). Thus using (3.15) total scattered radiation field is given in matrix form by

$$G^f = G_o \int_A C_o^{in} T^f F \ T^i \exp[ik_o(\bar{n}^f - \bar{n}^i)\cdot\bar{r}_s] D(\bar{r}_s) d\bar{A}\cdot\bar{n} \ G^i$$

$$\equiv G_o \ C(\bar{n}^f, \bar{n}^i) G^i \tag{3.36}$$

in which the matrix $T^f F \ T^i$ is given by

$$T^f F \ T^i = \begin{bmatrix} C_\psi^f(F^{VV}C_\psi^i - F^{VH}S_\psi^i) - S_\psi^f(F^{HV}C_\psi^i - F^{HH}S_\psi^i) & C_\psi^f(F^{VV}S_\psi^i + F^{VH}C_\psi^i) - S_\psi^f(F^{HV}S_\psi^i + F^{HH}S_\psi^i) \\ S_\psi^f(F^{VV}C_\psi^i - F^{VH}S_\psi^i) + C_\psi^f(F^{HV}C_\psi^i - F^{HH}S_\psi^i) & S_\psi^f(F^{VV}S_\psi^i + F^{VH}C_\psi^i) + C_\psi^f(F^{HV}S_\psi^i + F^{HH}C_\psi^i) \end{bmatrix} \tag{3.37}$$

For elements of the scattering surface where $\bar{n}$, $\bar{n}^i$ and $\bar{n}^f$ are coplanar ($[\bar{n}^f\ \bar{n}^i\ \bar{n}] = 0$), the cross-polarization terms $F^{VH}$ and $F^{HV}$ vanish. If in addition $\bar{n}^f, \bar{n}^i, \bar{n}$ and $\bar{a}_y$ are coplanar and $S^f_\psi = S^i_\psi = 0$, the matrix $T^f\ F\ T^i$ becomes diagonal and the cross-polarization terms $C^{VH}$ and $C^{HV}$ vanish (Bahar, 1980). Note however, that $F^{PQ} \neq 0$ and $C^{PQ} \neq 0$ for $P \neq Q$ even if $\sin(\phi^f - \phi^i) = 0$.

The integrand in (3.36) is finite as long as the area of the scattering surface ($A = \int d\bar{A} \cdot \bar{n}$) is finite. Thus the expression (3.36) is valid even as $(\bar{n} \cdot \bar{a}_y) = \cos\gamma \to 0$ in (3.17).

Since the full wave analysis accounts for upward and downward scattering with respect to the reference plane, it can also be used to determine the radiation fields due to multiple scattering. Thus, if the distance between regions of the scatterer (or scatterers) that contribute to multiple scattering is sufficiently large to justify far field techniques (steepest descent or stationary phase methods), the radiation field initially scattered in the direction $\bar{n}^f$ is regarded as the incident field in the evaluation of the multiply scattered field (see Fig. 3.2).

Note that while the shadow function $D_s(\bar{r}_s) = 0$ or $1$ (3.35), the elements of the scattering matrix F vanish in a continuous manner as the observer enters the shadow region (3.31) and the total radiation field $G^f$, (3.36), varies continuously as observer moves across the shadow boundary.

## 3.5 Invariance Properties of the Full Wave Solutions and the Duality and Reciprocity Relationships

In order to show that the full wave solution for the scattered radiation field is invariant to coordinate transformation, the different coefficients appearing in (3.36) are examined in groups. The incremental

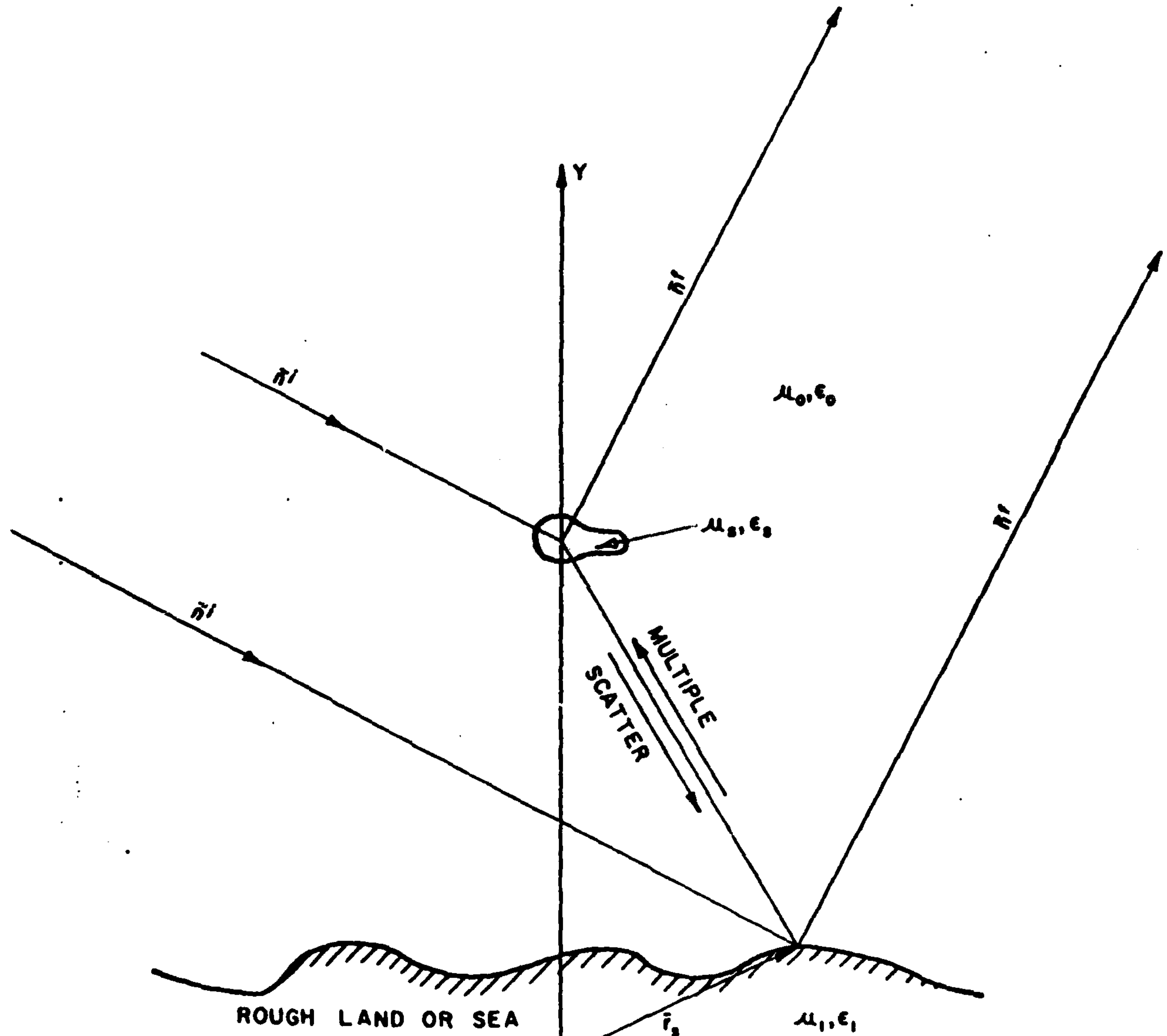



**Figure 3.2** Scattering by an irregularly shaped object in the vicinity of rough land or sea.

surface area of the scatterer is $dA = d\bar{A}\cdot\bar{n}$, (3.17). It is obviously invariant to coordinate transformation.

In the far field the distance from the source to points on the surface of the scatterer is

$$|r^i\bar{n}^i + \bar{r}_s| = r^i + \bar{r}_s\cdot\bar{n}_i \tag{3.38a}$$

Thus $T^iG^i \exp[-ik_o\, \bar{n}^i\cdot\bar{r}_s]$ is the expression for the incident (unperturbed) far field at the surface of the scatterer decomposed into vertically and horizontally polarized components with respect to the local tangent plane. (See Appendix 3.A) It is invariant to coordinate transformation in view of (3.12).

Similarly, the distance from points on the surface of the scatterer to the observation point in the far field is

$$|r^f\,\bar{n}^f - \bar{r}_s| = r^f - \bar{r}_s\cdot\bar{n}^f \tag{3.38b}$$

Thus the expression $G_o \exp[ik_o\,\bar{n}^f\cdot\bar{r}_s]$ is also invariant to coordinate transformation.

Since the expressions for $C_o^{in}\,F^{PQ}$ (3.25) depend upon the angles of incidence and scatter with respect to the local tangent plane only, it is also invariant to coordinate transformation. Finally, in view of the invariance of the transformation (3.14), the expression for the total scattered radiation field (3.36) is invariant under coordinate transformation.

Corresponding to the transformations

$$\bar{E}\to\bar{H},\ \bar{H}\to-\bar{E},\ \bar{J}\to\bar{M},\ \bar{M}\to-\bar{J},\ \mu\leftrightarrow\varepsilon \tag{3.39a}$$

the full wave solutions (3.36) can be shown to satisfy the following duality relationships:

$$F^{VV}(\varepsilon,\mu) = F^{HH}(\mu,\varepsilon)\ ,\ F^{HV}(\varepsilon,\mu) = -F^{VH}(\mu,\varepsilon) \tag{3.39b}$$

To examine the reciprocity relationships the locations of the source and observation points are interchanged. Thus, under the transformation

$$\bar{n}^i \leftrightarrow -\bar{n}^f \;,\; \bar{r}^i \leftrightarrow -\bar{r}^f \tag{3.40}$$

the following relationships can be shown to be satisfied

$$T^i(-\bar{n}^f) = T^f(\bar{n}^f) = T^f(\bar{n}^f)^{T*} \tag{3.41a}$$

$$T^f(-\bar{n}^i) = T^i(\bar{n}^i) = T^i(\bar{n}^i)^{T*} \tag{3.41b}$$

$$F(-\bar{n}^i,-\bar{n}^f) = F(\bar{n}^f,\bar{n}^i)^{T*} \tag{3.41c}$$

in which the superscript T denotes the transpose of the matrix and the symbol * denotes that the nondiagonal elements of the matrix are multiplied by -1. Thus on noting that for any 2x2 matrices X and Y

$$X^* Y^* = (XY)^* \tag{3.42}$$

the following relationship for $C(\bar{n}^f,\bar{n}^i)$, (3.36) is obtained (Bahar, 1981a),

$$C(-\bar{n}^i,-\bar{n}^f) = [C(\bar{n}^f,\bar{n}^i)]^{T*} \tag{3.43}$$

and the total scattered radiation fields $G^f$, (3.36) satisfy the reciprocity relationships in electromagnetic theory.

## 3.6 High Frequency Approximations of the Full Wave Solution--Comparison with Physical Optics Solutions

At very high frequencies the major contributions to the scattered radiation fields (3.36) come only from regions of the rough surface where the phase of the expression $\exp[ik_o\bar{r}_s\cdot(\bar{n}^f-\bar{n}^i)]$ is stationary. This phase is constant when

$$\begin{aligned}\nabla[\bar{r}_s\cdot(\bar{n}^f-\bar{n}^i)] &= \nabla[(\bar{r} \mp f\bar{a}_y)\cdot(\bar{n}^f-\bar{n}^i)]\\ &= (\bar{n}^f-\bar{n}^i) \mp (\bar{n}^f-\bar{n}^i)\cdot\bar{a}_y\,\nabla f = 0\end{aligned} \tag{3.44}$$

Thus at the stationary phase points, the unit vector $\bar{n}_s$ (normal to the surface $f(x,y,z) = 0$ (3.22a), is in the direction $\bar{n}^f-\bar{n}^i$ and

$$\bar{n} = \bar{n}_s = \frac{\bar{n}^f - \bar{n}^i}{|\bar{n}^f - \bar{n}^i|} = \sin\gamma_s \cos\delta_s \, \bar{a}_x + \cos\gamma_s \, \bar{a}_y + \sin\gamma_s \sin\delta_s \, \bar{a}_z$$

$$\equiv \pm(-h_{xs} \, \bar{a}_x + \bar{a}_y - h_{zs} \, \bar{a}_y)\cos\gamma_s \tag{3.45}$$

The incident and scatter angles $\theta_o^s$ with respect to the tangent plane at the stationary points are

$$\cos\theta_o^{ins} = -(\bar{n}^i \cdot \bar{n}_s) = \frac{1 - \bar{n}^f \cdot \bar{n}}{|\bar{n}^f - \bar{n}^i|} = \left[\frac{1 - \bar{n}^f \cdot \bar{n}^i}{2}\right]^{\frac{1}{2}} = \left[\frac{1 + C_o^i C_o^f - S_o^i S_o^f \cos(\phi^f - \phi^i)}{2}\right]^{\frac{1}{2}} \tag{3.46a}$$

and

$$\cos\theta_o^{fns} = (\bar{n}^f \cdot \bar{n}_s) = \cos\theta_o^{ins} \equiv \cos\theta_o^s \equiv C_o^s, -n^f \cdot n^i = \cos 2\theta_o^s \tag{3.46b}$$

The corresponding angles $\theta_1^{ins}$ and $\theta_1^{fns}$ for medium $\varepsilon_1, \mu_1$, are obtained through Snell's law. Thus

$$S_1^s = S_o^s / n_r \text{ and } C_1^s = [1 - (S_1^s)^2]^{\frac{1}{2}} \tag{3.47}$$

Furthermore at the stationary points

$$\cos(\phi^{fns} - \phi^{ins}) = \frac{(\bar{n}^f \times \bar{n}_s) \cdot (\bar{n}^i \times \bar{n}_s)}{|\bar{n}^f \times \bar{n}_s| \, |\bar{n}^i \times \bar{n}_s|} = 1 \tag{3.48a}$$

$$\sin(\phi^{fns} - \phi^{ins}) = \frac{[\bar{n}_s \; \bar{n}^f \; \bar{n}^i]}{|\bar{n}^f \times \bar{n}_s| \, |\bar{n}^i \times \bar{n}_s|} = 0 \tag{3.48b}$$

$$\cos\gamma_s = \bar{n}_s \cdot \bar{a}_y = (C_o^i + C_o^f)/2 \, C_o^s \tag{3.49a}$$

$$\sin\delta_s = \frac{S_o^f \sin\phi^f - S_o^i \sin\phi^i}{[(2 \, C_o^s)^2 - (C_o^i + C_o^f)^2]^{\frac{1}{2}}}, \quad \cos\delta_s = \frac{S_o^f \cos\phi^f - S_o^i \cos\phi^i}{[(2 \, C_o^s)^2 - (C_o^i + C_o^f)^2]^{\frac{1}{2}}} \tag{3.49b}$$

The angles $\psi^{is}$ and $\psi^{fs}$ are given by

$$C_\psi^{is} = \cos\psi^{is} = \frac{C_o^i + C_o^f - C_o^i(1 + \cos 2\theta_o^s)}{S_o^i \sin 2\theta_o^s}, \quad S_\psi^{is} = \sin\psi^{is} = \frac{S_o^f \sin(\phi^f - \phi^i)}{\sin 2\theta_o^s} \tag{3.50a}$$

$$C_\psi^{fs} = \cos\psi^{fs} = \frac{C_o^f + C_o^i - C_o^f(1 + \cos 2\theta_o^s)}{S_o^f \sin 2\theta_o^s}, \quad S_\psi^{fs} = \sin\psi^{fs} = \frac{S_o^i \sin(\phi^f - \phi^i)}{\sin 2\theta_o^s} \tag{3.50b}$$

Thus for the stationary points on the scatterer

$$F^{PQ}(\bar{n}^{fn}, \bar{n}^{in}) \to 0 \quad P \neq Q \tag{3.51a}$$

and

$$\frac{C_o^{in} F^{PP}(\bar{n}^{fn},\bar{n}^{in})}{\bar{n}\cdot\bar{a}_y} \quad \frac{C_o^s R_{1o}^P(C_o^s)}{\cos\gamma_s} = \frac{2\cos^2\theta_o^s R_{1o}^P(\theta_o^s)}{C_o^f + C_o^i} = C_o^i R_{1o}^{Ps} F_o(\bar{n}^f,\bar{n}^i) \tag{3.51b}$$

where $R_{1o}^{Ps} = R_{1o}^P(\theta_o^s)$ is the Fresnel reflection coefficient for vertically or horizontally polarized waves (P=V,H) incident at an angle $\theta_o^s$ upon the plane normal to $\bar{n}_s$. The function

$$F_o(\bar{n}^f,\bar{n}^i) = \frac{\cos^2\theta_o^s}{C_o^i \cos\left(\frac{\theta_o^f+\theta_o^i}{2}\right)\cos\left(\frac{\theta_o^f-\theta_o^i}{2}\right)} = \frac{1+C_o^i C_o^f - S_o^i S_o^f \cos(\phi^f-\phi^i)}{C_o^i(C_o^i+C_o^f)} \tag{3.52}$$

is exactly equal to the function $F_3(\theta_o^i,\theta_o^f,\phi^f-\phi^i)$ derived by Beckmann and Spizzichino (1963) using a Physical Optics approach for perfectly conducting rough surfaces. Thus if there are points on the illuminated portion of the surface of the scatterer for which $\bar{n} = \bar{n}_s$, the Physical Optics limit for the scattered radiation field can be obtained from the full wave solutions by replacing the expression for $C_o^{in} T^f F T^i$ in (3.36) by its value at the stationary points and by replacing the shadow function $D(\bar{r})$ by unity. Thus in the Physical Optics limit $C_o^{in} T^f F T^i$ can be factored out of the integral (3.36) and the expression for C reduces to

$$C = C_o^i F_o I \begin{bmatrix} C_\psi^{fs} R_{1o}^{Vs} C_\psi^{is} + S_\psi^{fs} R_{1o}^{Hs} S_\psi^{is} & C_\psi^{fs} R_{1o}^{Vs} S_\psi^{is} - S_\psi^{fs} R_{1o}^{Hs} C_\psi^{is} \\ S_\psi^{fs} R_{1o}^{Vs} C_\psi^{is} - C_\psi^{fs} R_{1o}^{Hs} S_\psi^{is} & S_\psi^{fs} R_{1o}^{Vs} S_\psi^{is} + C_\psi^{fs} R_{1o}^{Hs} C_\psi^{is} \end{bmatrix} \tag{3.53a}$$

in which the integral I is given by

$$I(\bar{n}^f,\bar{n}^i,\bar{r}_s,A) = \int_A \exp[ik_o(\bar{n}^f-\bar{n}^i)\cdot\bar{r}_s]d\bar{A}\cdot\bar{a}_y \tag{3.53b}$$

For scatterers that are very good conductors, assuming $C_o^s \neq \pi/2$; $R_{1o}^{Vs} \to 1$ and $R_{1o}^{Hs} \to -1$, (3.53a) reduced to

$$C = C_o^i F_o I \begin{bmatrix} \cos(\psi^{fs}+\psi^{is}) & \sin(\psi^{fs}+\psi^{is}) \\ \sin(\psi^{fs}+\psi^{is}) & -\cos(\psi^{fs}+\psi^{is}) \end{bmatrix} = C_o^i F_o I U^s \tag{3.54}$$

Thus the matrix $U^s$ in (3.54) is unitary.

## 3.7 Realizability and the Backscatter Cross-Sections

For a perfectly conducting plane of area A centered at the origin and normal to the vector $\bar{n} = \bar{n}_s$, the shadow function $D(\bar{r}_s)$ reduces to unity and $(\bar{n}^f - \bar{n}^i)\cdot\bar{r}_s = 0$. Thus

$$I(\bar{n}^f, \bar{n}^i, \bar{r}_s, A) = A\,\bar{n}_s\cdot\bar{a}_y = A_y \tag{3.55a}$$

in which $A_y$ is the projection of the area $\bar{A}$ on the reference plane. From (3.51b) it follows that

$$C_o^i F_o = \frac{C_o^s}{\cos\gamma_s} = \frac{-\bar{n}^i\cdot\bar{n}_s}{\bar{n}_s\cdot\bar{a}_y} \tag{3.55b}$$

For this case the scattering matrix C (3.54) is therefore given by

$$C(\bar{n}^f, \bar{n}^i) = A\cos\theta_o^s\, U^s \tag{3.56}$$

Since $U^s$ is a unitary matrix (3.54), energy is conserved independent of the angle of incidence and the polarization of the waves incident upon the conducting plane. This realizability relationship is not satisfied by the iterative or perturbational solutions (Bahar, 1981a).

The backscattered radiation fields are given by (3.36) with $\bar{n}^f = -\bar{n}^i$. This quantity is maximum for a perfectly conducting plane of area A centered at the origin and normal to $\bar{n} = \bar{n}_s = -\bar{n}^i = \bar{n}^f$. For this case the backscattered fields are obtained from (3.56) on substituting $\theta_o^s = 0$. Since in addition $\psi^{fs} + \psi^{is} = 0$, the unitary matrix reduces to a diagonal matrix. Thus

$$\left|[C^{PQ}(-\bar{n}^i, \bar{n}^i)]_{max}\right| = A\delta_{PQ} = \frac{A_y}{\cos\theta_o}\,\delta_{PQ} \tag{3.57}$$

where $\delta_{PQ}$ is the Kronecker delta. Thus the maximum backscattered power density is proportional to $A^2$, the square of the area of the perfectly

conducting plane which is perpendicular to the incident wave normal $\bar{n}^i$. In (3.57) $\theta_o$ is the angle of incidence or scatter with respect to the reference plane (y = 0).

The normalized backscatter cross section for a scatterer of arbitrary shape is defined here as

$$\sigma^{PQ} = [|C^{PQ}(-\bar{n}^i,\bar{n}^i)|/\int D(\bar{r}_s)d\bar{A}\cdot\bar{n}]^2 \tag{3.58}$$

The quantity in the denominator of (3.58) is the area of the illuminated surface of the scatterer. Thus the maximum value of the normalized, dimensionless quantity $\sigma^{PQ}$ defined in (3.58) is unity.

When the illuminated portion of the surface of the scatterer is slightly rough such that $\bar{n} \simeq \bar{a}_y$ the perturbational approach is valid. Thus the full wave solutions for the backscattered cross sections reduce to the perturbational solutions on replacing $\bar{n}$ by $\bar{a}_y$ in the expressions (3.25) and on ignoring the shadow function $D(\bar{r}_s)$ (Barrick, 1970). In this case the expression for C (3.36) simplifies considerably since $C_o^{in}T^f \; F \; T^i$ becomes independent of the variable of integration and

$$C \rightarrow (C_o^i T^f \; F \; T^i)_{\bar{n}=\bar{a}_y} \cdot I(\bar{n}^f, n^i, \bar{r}_s, A)$$

$$\equiv C_o \; I(\bar{n}^f,\bar{n}^i,\bar{r}_s,A) \tag{3.59}$$

where the integral I is defined by (3.53b) and the matrix $C_o$ is defined by (3.59). Similarly, if there are points on the illuminated portion of the surface of the scatterer for which $\bar{n} = \bar{n}_s$, at high frequencies one can use the Physical Optics approximations derived in Section 3.6. In this case $\bar{n}$ is replaced by $\bar{n}_s$ in (3.36) and assuming that the specular points are in the visible portion of the surface of the scatterer, $D(\bar{r}_s)$ is set equal to unity. Thus for the high frequency limit the expression for C (3.36) also simplifies significantly. The expression for $C_o^{in}T^f \; F \; T^i$ becomes independent of the variable of integration and

$$C \rightarrow (C_o^i\ T^f\ F\ T^i)_{\bar{n}=\bar{n}_s} \cdot I(\bar{n}^f, \bar{n}^i, \bar{r}_s, A)/\bar{n}\cdot\bar{a}_y$$

$$\equiv C_\infty\ I(\bar{n}^f, \bar{n}^i, \bar{r}_s, A) \tag{3.60}$$

where $C_\infty$ is defined by (3.60).

Thus for these special cases (small slope or high frequency) the expression for the normalized backscatter cross section (3.58) reduced to

$$\sigma_{\infty}^{PQ} = |C_{\infty}^{PQ}|^2 \left|\int_A \exp[-2i\bar{n}^i\cdot\bar{r}_s]dxdz\right|^2/[\int_A D(\bar{r}_s)d\bar{A}\cdot n]^2$$

$$= |C_{\infty}^{PQ}|^2 \iint_{AA'} \exp[-2i\bar{n}^i\cdot(\bar{r}_s - \bar{r}_s')]dx'dz'dxdz/[\int_A D(\bar{r}_s)d\bar{A}\cdot\bar{n}]^2 \tag{3.61}$$

in which

$$\bar{r}_s' = x'\bar{a}_x + h(x',y')\bar{a}_y + z'\bar{a}_z \tag{3.62}$$

Since the full wave solutions for the backscatter cross section $\sigma^{PQ}$ (3.58) reduce to the perturbational form $\sigma_o^{PQ}$ for slightly rough surfaces (Barrick, 1970) and to the Physical Optics form $\sigma_\infty^{PQ}$ when the stationary phase approximations are used, the full wave solutions provide a direct connection between these special forms for the backscatter cross section and reconcile the significant differences between them. Moreover, the full wave method can be applied to a wide class of problems that cannot be solved by either the perturbational or the Physical Optics approach.

## 3.8 Properties of the Full Wave Solutions at Grazing, Specular and Brewster Angles and Applications to Random and Periodic Rough Surfaces

For grazing incident or scatter angles, with respect to the local tangent planes, (normal to the vector $\bar{n}$) all the full wave expressions for $C_o^{in}F^{PQ}(\bar{n}^f,\bar{n}^i)$ (3.25) vanish. The corresponding expressions in the perturbational solutions vanish only for grazing incident angles with respect to the reference plane (normal to $\bar{a}_y$). On the other hand, the Physical Optics expression for (3.25) vaniehs only for grazing angles with respect to the stationary phase plane where $n = \bar{n}_s$. The property

of full wave solutions at grazing angles with respect to the local tangent plane is responsible for the fact that the full wave solutions for the scattered radiation fields vanish in a continuous manner as the observer moves across a shadow boundary. For points on the irregular boundary where the local tangent plane coincides with the stationary phase plane normal to $\bar{n} = \bar{n}_s$ (locally specular scatter), the full wave expressions for $F^{PQ}$ reduce to

$$\left.\begin{aligned} F^{PP}(\bar{n}^{fn},\bar{n}^{in}) &= R^{PP}(\theta_o^s) \quad , \; P=V,H \\ F^{PQ}(\bar{n}^{fn},\bar{n}^{in}) &= 0 \quad , \qquad P\neq Q \end{aligned}\right\} \bar{n} = \bar{n}_s \tag{3.63}$$

The corresponding perturbational solutions for $F^{PP}$ reduce to $R^{PP}(\theta_o^s)$ only for specular angles with respect to the reference plane normal to $\bar{a}_y$. Similarly, the perturbational solutions for $F^{PQ}(P\neq Q)$ vanish only for specular angles with respect to the fixed reference plane.

For the specular case with respect to the reference plane the Physical Optics solution for $F^{PP}$ reduces to $R^{PP}(\theta_o^s)$ for the entire rough surface including those portions that are not specularly oriented with respect to transmitter and receiver. Furthermore, while the Physical Optics approach does not provide any expression corresponding to $F^{PQ}(P\neq Q)$, the full wave expression for $F^{PQ}$ vanishes only at the specular points on the irregular boundary. It is interesting to note that if $\theta_o^s$ (the incident or scatter angle with respect to the stationary phase plane) is equal to the Brewster angle $\theta_o^B$, the Physical Optics solution for the scattered radiation field vanishes. Similarly, if the incident on scatter angles with respect to the reference plane (normal to $\bar{a}_y$) equals the Brewster angle, the perturbational solution also vanishes. The full wave solution on the other hand does not vanish. The integrand

in the expression for the full wave solutions (3.36) vanishes only at the stationary phase points where $\theta_o^i = \theta_o^f = \theta_o^B$. Thus, the full wave solutions for scattering by rough surfaces does not vanish even if the incident and scatter angles with respect to the stationary phase plane (normal to $\bar{n}_s$) or with respect to the reference plane (normal to $\bar{a}_y$) are equal to the Brewster angle.

For very good conducting boundaries with $|\epsilon_r| >> 1$ and $\mu_r = 1$ ($|\eta_r| << 1$), the expressions for $C_o^{in}F^{PQ}$ (3.25) simplify considerably.

$$C_o^{in}F^{VV} = 2[\cos(\phi^{fn}-\phi^{in}) - S_o^{in}S_o^{fn}]/(C_o^{in}+C_o^{fn}) \tag{3.64a}$$

$$C_o^{in}F^{HH} = -2C_o^{in}C_o^{fn}\cos(\phi^{fn}-\phi^{in})/(C_o^{in}+C_o^{fn}) \tag{3.64b}$$

$$C_o^{in}F^{HV} = -2\sin(\phi^{fn}-\phi^{in})C_o^{fn}/(C_o^{in}+C_o^{fn}) \tag{3.64c}$$

$$C_o^{in}F^{VH} = -2\sin(\phi^{fn}-\phi^{in})C_o^{in}/(C_o^{in}+C_o^{fn}) \tag{3.64d}$$

With the exception of $F^{HH}$ the above expressions cannot be used for grazing angles if the boundary is highly conducting. In general, the approximate formulas for $C_o^{in}F^{PQ}$ (3.64) must be restricted not only to good conducting boundaries but also to angles of incidence or scatter that are less than the pseudo-Brewster angles $\theta_o^B$. For vertically polarized waves

$$C_o \to C_o^B = \cos\theta_o^B = \eta_r C_1^B = [(\eta_r^2-1)/\epsilon_r^2-1]^{\frac{1}{2}} \to (\epsilon_r+1)^{-\frac{1}{2}} \to \epsilon_r^{-\frac{1}{2}}. \tag{3.65}$$

Thus (3.64) can be used only if $C_o^{in}$ and $C_o^{fn}$ are greater than $(\epsilon_r)^{-\frac{1}{2}}$.

For perfectly conducting boudaries, the steepest descent method, used in evaluating the radiation fields at grazing angles, needs to account for the poles in the vicinity of the saddle points. Thus the apparent singularity in the expression for the fields at grazing angles over perfectly conducting surfaces is removed (Bahar, 1981b).

The full wave solutions (3.36) can be applied to periodic structures by multiplying the expressions for the scattered radiation fields due

to a single element of the periodic rough surface by two dimensional array factors (Bahar, 1980). Implicit in these array factors are the expressions for the grating angles.

The expressions for the full wave solutions depend upon the profile $h(x,z)$ of the rough surface and upon its gradient. Thus in order to determine the statistical average of $C^{PQ}$ (3.36) and its variance, $\langle C^{PQ}\rangle$ and $D\{C^{PQ}\}$ respectively, it is necessary in general to know the statistics of the random variables $h$, $h_x$ and $h_z$. For two special cases (the small slope--perturbational solution and the stationary phase--geometrical optics solution) considered in this work, the statistical average and the variance of the scattered radiation fields can be expressed in terms of the one and two dimensional characteristic functions respectively (Bahar, 1981a).

## 3.9 Concluding Remarks

The full wave approach is applied in this paper to the problem of depolarization of the scattered radiation field by an object of irregular shape and finite conductivity. The principal elements of the full wave approach are outlined in the introduction (Section 3.3). The full wave solutions are presented in a form that can be readily compared with earlier solutions (Section 3.4). They are used to resolve the discrepancies between the earlier solutions and to bridge the wide gap that exists between them. Realizability, reciprocity and duality relationships in electromagnetic theory are examined and the full wave solutions are shown to be invariant to coordinate transformations.

The full wave approach can also be used to determine the scattered radiation fields due to lateral variations in the permittivity $\varepsilon$ and

permeability $\mu$ (Bahar, 1973a,b). Thus they can be applied to mixed path propagation problems.

Since the full wave approach has been generalized to inhomogeneous multilayered structures of arbitrarily varying thickness, it can also be applied to problems in which irregularly shaped objects are imbedded in the earth's crust. If the medium of the scatterer is nondissipative, transmission through the scatterer can also be accounted for in the analysis.

When the transmitter or receiver are near the scatterer, the lateral wave or surface wave terms of the full wave expansions could be significant especially near a shadow region. In these cases the full wave approach can be used to determine the coupling between the radiation term and the lateral and surface wave terms (Bahar, 1977).

## 3.10 References

1. Bahar, E., "Depolarization of Electromagnetic Waves Excited by Distribution of Electric and Magnetic Sources in Inhomogeneous Multilayered Structures of Arbitrarily Varying Thickness--Generalized Field Transforms," Journal of Mathematical Physics, 14(11), pp. 1502-1509, 1973a.
2. Bahar, E., "Depolarization of Electromagnetic Waves Excited by Distribution of Electric and Magnetic Sources in Inhomogeneous Multilayered Structures of Arbitrarily Varying Thickness--Full Wave Solution," Journal of Mathematical Physics, 14(11), pp. 1510-1515, 1973b.
3. Bahar, E., "Coupling Between Guided Surface Waves, Lateral Waves and the Radiation Fields by Rough Surfaces--Full Wave Solutions," IEEE Transactions on Microwave Theory and Techniques, MTT-35(11), pp. 923-931, 1977.

4. Bahar, E., "Full Wave and Physical Optics Solutions for Scattered Radiation Fields by Rough Surfaces--Energy and Reciprocity Relationships," IEEE Transactions on Antennas and Propagation, AP-26(4), pp. 603-614, 1978.

5. Bahar, E., "Full Wave Solutions for the Scattered Radiation Fields from Rough Surfaces with Arbitrary Slope and Frequency," IEEE Transactions on Antennas and Propagation, AP-28(1), pp. 11-21, 1980.

6. Bahar, E., "Full Wave Solutions for the Depolarization of the Scattered Radiation Fields by Rough Surfaces of Arbitrary Slope," IEEE Transations on Antennas and Propagation, AP-29(3), pp. 443-454, 1981a.

7. Bahar, E., "Scattering and Depolarization by Rough Surfaces Near Grazing Angles--Full Wave Solutions," IEEE Transactions on Antennas and Propagation, in press, 1981b.

8. Bahar, E. and R. Rajan, "Depolarization and Scattering of Electromagnetic Waves by Irregular Boundaries of Arbitrary Incident and Scatter Angles--Full Wave Solutions," IEEE Transactions on Antennas and Propagation, AP-27(2), pp. 214-225, 1979.

9. Barrick, D. E., "Rough Surfaces" Chapter 9 in the Radar Cross Section Handbook, Plenum Press, New York, 1970.

10. Beckmann, P., The Depolarization of Electromagnetic Waves, Chapter 3, Golem Press, Boulder, Colorado, 1968.

11. Beckmann, P. and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, Chapter 3, MacMillan, New York, 1963.

12. Rice, S. O., "Reflection of Electromagnetic Waves from Slightly Rough Surfaces," Communications of Pure and Applied Math, 4, pp. 351-378, 1951.

## 3.A Appendix

The relationship between the reference coordinate system $X(x,y,z)$ and the variable local coordinate system $\bar{X}(\bar{x},\bar{y},\bar{z})$ associated with the local tangent plane normal to $\bar{n}$ (3.22) can be expressed as follows:

$$\bar{X} = \begin{pmatrix} \bar{x}^1 \\ \bar{x}^2 \\ \bar{x}^3 \end{pmatrix} = A \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix} \qquad (3A.1)$$

Thus, $\bar{x} = a_{ij}x^j$ (sum on $j = 1,2,3$)

where $x^1 = x$, $x^2 = y$, $x^3 = z$, etc. The elements $a_{ij}$ of the 3x3 transformation matrix A are

$$a_{ij} = \bar{n}_i \cdot \bar{a}_j \qquad i,j = 1,2,3. \qquad (3A.2)$$

where $\bar{a}_1 = \bar{a}_x$, $\bar{a}_2 = \bar{a}_y$ and $\bar{a}_3 = \bar{a}_z$ are the unit vectors of the reference coordinate system and $\bar{n}_i$ are unit vectors normal to the coordinate surfaces of the local coordinate system. Thus

$$\bar{n}_1 = \bar{n} \times (\bar{a}_1 \times \bar{n})/|\bar{a}_1 \times \bar{n}| \qquad (3A.3)$$

$$\bar{n}_2 = \bar{n} \qquad (3A.4)$$

$$\bar{n}_3 = (\bar{a}_1 \times \bar{n})/|\bar{a}_1 \times \bar{n}| \qquad (3A.5)$$

Thus the unit vectors $\bar{n}_1$, $\bar{n}_2$ and $\bar{n}_3$ are orthogonal. The vector $\bar{n}_2$ is normal to the local tangent plane (3.22) while $\bar{n}_1$ and $\bar{n}_3$ lie in the local tangent plane. For a horizontal surface $h(x,z) = \text{const.}$, the unit vectors $\bar{n}_i$ are equal to $\bar{a}_i$ and A reduces to the identity matrix. In general the transformation matrix A is hermetian, thus the determinant of A is equal to unity and

$$A^{-1} = A^T \qquad (3A.6)$$

# 4.0 SCATTERING CROSS SECTIONS FOR COMPOSITE RANDOM ROUGH SURFACES --FULL WAVE ANALYSIS

## 4.1 Background

In this work the full wave approach to rough surface scattering is applied to composite models of rough surfaces. It is shown that both specular point scattering as well as Bragg scattering are accounted for in the analysis in a self-consistent manner. The results are compared with earlier solutions based on a combination of Physical Optics and perturbation theories. Using the full wave approach it is not essential to decompose the rough surface into individual surfaces with different roughness scales unless it is desired to separate the specular point contribution from the Bragg contribution to the scattering cross sections. The decomposition of the rough surface not only enhances one's physical insight but also facilitates the numerical evaluation of the scattering cross sections. Shadowing is also accounted for in the analysis.

## 4.2 Discussion

Physical Optics and perturbation theories have been applied to problems of scattering of electromagnetic waves from rough surfaces $f(x,y,z)=y-h(x,z)=0$ (Valenzuela, 1968). However, these theories can only be applied to a limited class of rough surfaces. Thus perturbation theory (Rice, 1951; Barrick, 1970) can be applied to problems in which it is usually assumed that

$$k_o^2\langle h^2\rangle \ll 1,\ \partial h/\partial x = h_x \ll 1,\ \partial h/\partial z = h_z \ll 1 \qquad (4.1)$$

in which $\langle h^2\rangle$ is the mean square of the rough surface height and $k_o$ is the wave number of the electromagnetic wave. Physical Optics, which is based on the Kirchhoff approximations of the surface fields (Beckmann, 1968), is applicable to surface for which the radii of curvature of the

rough surface are large compared to the electromagnetic wavelength $\lambda$.

Wright (1966) and Semyonov (1966) apply the above theories to composite surfaces made up of irregularities that are both small as well as large compared to the wavelength $\lambda$. More recently Brown (1978, 1980) applied Physical Optics as well as Burrow's perturbation theory (1967) to derive the expression for the backscatter cross section from perfectly conducting rough surfaces in terms of a sum of two backscatterer cross sections. The first is the backscatter cross section for the surface with the large scale roughness $h_\ell$, while the second is the backscatter cross section associated with the small scale roughness $h_s$. In his work, Brown (1978, 1980) assumes that the radii of curvature of the surface, $h_\ell$, is larger than the wavelength $\lambda$ and $k_o^2\langle h_\ell^2\rangle >> 1$. In addition, he assumes that $h_s$ satisfies the conditions (4.1). Thus in his work the specification of the wavenumber $k_d$ (where spectral splitting is assumed to occur), is based upon the characteristics of the small scale structure ($k_o^2\langle h_s^2\rangle << 1$) rather than upon the characteristics of the large scale surface (Brown, 1978; Hagfors, 1966; Tyler, 1978). In Brown's analysis the backscatter cross section associated with the surface with the small scale roughness is expressed in terms of a two dimensional convolution of transforms.

In the composite models of Wright (1968), Semyonov (1966), and Valenzuela (1968) which are "mostly based on physical considerations" the rough surface is approximated by "patches" of slightly rough surfaces that ride the large waves. Thus in their work the scattering cross section associated with the surface with the small scale roughness is obtained by averaging over the distribution of slopes of the large scale surface roughness.

In this work the full wave approach to rough surface scattering is extended to composite models of the rough surface with moderate

mean square slopes (Bahar, 1981b,c). Since the full wave approach accounts for both specular point scattering as well as Bragg scattering in a self-consistent manner (Bahar, 1981b,c) without introducing perturbation and Physical Optics theories, it is not necessary to decompose the rough surface into two surfaces with large and small roughness scales, $h_\ell$ and $h_s$ respectively. Thus the need to specify $k_d$, the wavenumber where spectral splitting is assumed to occur, does not necessarily arise when the full wave approach is used. However, the decomposition of the rough surface enhances one's physical insight and also facilitates the numerical evaluation of the scattering cross sections.

In Section 4.3 the problem is formulated in terms of the full wave approach and the principal results from earlier analysis are summarized. In order to compare the full wave solutions for the scattering cross sections with earlier solutions (Valenzuela, 1968; Brown, 1978, 1980) it is also assumed in Section 4.4 that the composite surface can be decomposed into two statistically independent surfaces with large and small roughness scales $h_\ell$ and $h_s$. It is shown that the total scattering cross section can be written as a weighted sum of individual cross sections. The cross section associated with specular point scattering is multiplied by $|\chi^R|^2$ the magnitude squared of the characteristic function for the small scale surface height $h_R$ (4.30a). The cross section associated with Bragg scattering is expressed in terms of an integral over the slopes of the rough surface (4.44) as predicted on the basis of physical considerations (Valenzuela, 1968). This term is shown to be in agreement with perturbation theory (Rice, 1951; Barrick, 1970). The integrand in this term is proportional to

the surface height spectral density $W(v_{\bar{x}}, v_{\bar{z}})$ whose arguments are the two components $v_{\bar{x}}$ and $v_{\bar{z}}$ of the fector $\bar{v} = k_o(\bar{n}^f - \bar{n}^i)$ in the local tangent plane ($\bar{n}^f$ and $\bar{n}^i$ are unit vectors in the direction of the scattered and incident wave normals). The full wave solution can also be expressed in terms of the rough surface tilts "in" and "perpendicular" to the plane of incidence and thereby compared with the results of Valenzuela, et al. (Valenzuela, Lang and Daley, 1971). It is also shown that since Brown (1978, 1980) assumes that the small scale surface height correlation function is a function of distance in the reference (mean) plane and not distances along the large scale surface as in this work and implicitly in Valenzuela's work, the full wave results are in agreement with Brown's results only if the large scale surface roughness has small mean square slopes. Shadowing is accounted for explicitly in this work as in the work by Brown (1978, 1980).

Throughout this work an $\exp(i\omega t)$ time dependence is assumed.

## 4.3 Formulation of the Problem

For the incoherent, diffuse field, the scattering cross section per unit area is defined as (Ishimaru, 1978),

$$\langle\sigma^{PQ}\rangle = 4\pi(r^f)^2\langle|E^{Pf} - \langle E^{Pf}\rangle|^2\rangle/A_y|E^{Qi}|^2 \quad , \; P,Q=V,H \qquad (4.2)$$

in which the first superscript P=V,H indicates the polarization (vertical or horizontal) of the scattered field, $E^{Pf}$ and the second superscript Q=V,H indicates the polarization of the incident field, $E^{Qi}$. The projection of the area of the rough surface A on the reference plane normal to $\bar{a}_y$ is $A_y$ and $r^f$ is the distance from the origin (associated with the rough surface) and the observation point in the far field (see Fig. 4.1). Using the full wave solutions for the scattered radiation fields (Bahar, 1981a), the expression for the scattering cross section

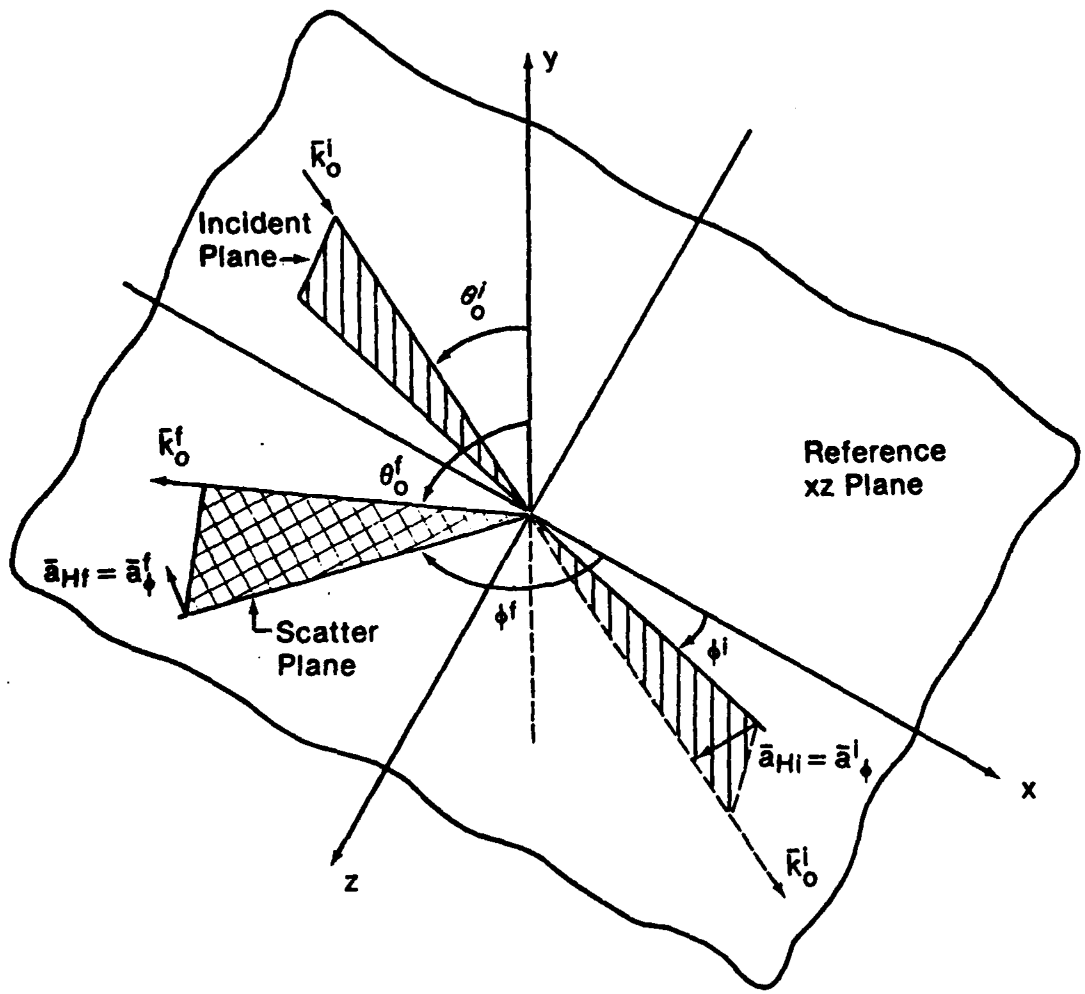



Figure 4.1 Plane of incidence, scatter plane and mean (reference) plane.

for random rough surfaces is given by (Bahar, 1981b)

$$\langle\sigma^{PQ}\rangle = \frac{k_o^2}{\pi A_y} \left\langle \int \left[ S^{PQ} \exp[iv_y(h-h')] - \left| \left\langle \frac{D^{PQ}U}{\bar{n}\cdot\bar{a}_y} \exp(iv_y y) \right\rangle \right|^2 \right] \right.$$

$$\left. \cdot \exp[iv_x(x-x')+iv_z(z-z')]dxdz\, dx'dz' \right\rangle \tag{4.3}$$

in which $k_o$ is the free space wavenumber. The vector $\bar{v}$ in the cartesian coordinate system is

$$\bar{v} = k_o(\bar{n}^f - \bar{n}^i) = v_x\, \bar{a}_x + v_y\, \bar{a}_y + v_z\, \bar{a}_z \tag{4.4}$$

in which $\bar{n}^i$ and $\bar{n}^f$ are unit vectors in the direction of the incident and scattered waves respectively.

The characteristic function $\chi(v_y)$ is given by

$$\chi(v_y) = \int_{-\infty}^{\infty} \exp(iv_y h)p(h)dh \tag{4.5}$$

in which $p(h)$ is the surface height probability density function. The coefficient $S^{PQ}$ is

$$S^{PQ} = \frac{D^{PQ}(\bar{r})U(\bar{r})}{(\bar{n}\cdot\bar{a}_y)} \frac{D^{PQ*}(\bar{r}')U(\bar{r}')}{(\bar{n}'\cdot\bar{a}_y)} + \left| \frac{D^{PQ}(\bar{r})U(\bar{r})}{\bar{n}\cdot\bar{a}_y} \right|^2 \tag{4.6}$$

in which $D^{PQ}(\bar{r})$ depends upon the polarization (P,Q=V,H) and directions ($\bar{n}^i$ and $\bar{n}^f$) of the incident and scattered waves, the unit vector $\bar{n}$ normal to the rough surface and the electromagnetic parameters of the medium of propagation and the scatterer (Bahar, 1981b). The shadow function $U(\bar{r})$ is unity when the surface is both illuminated by the source and visible at the observation point and zero otherwise (Sancer, 1969). The symbol < > denotes the statistical average. Thus assuming that either the rough surface radii of curvature are large compared to a wavelength or the rough surface slope is small and that the rough surface heights and slopes are statistically independent (Bahar, 1981c),

$$\langle S^{PQ}[\exp iv_y(h-h') - |\chi(v_y)|^2]\rangle \to$$

$$\int\left|\frac{D^{PQ}(\bar{r})U(\bar{r})}{\bar{n}\cdot\bar{a}_y}\right|^2[\exp\{iv_y(h-h')\}-|\chi(v_y)|^2]p(\bar{n},U)W(h,h')dhdh'dh_x dh_z dU \tag{4.7}$$

The joint rough surface height probability density function is $W(h,h')$ and the joint probability density function $p(\bar{n},U)$ can be expressed in terms of the conditional density $p(U|\bar{n})$ (Sancer, 1969)

$$p(\bar{n},U) = p(\bar{n})p(U|\bar{n}) \tag{4.8}$$

in which $p(\bar{n})$ is the density function of the gradient of the rough surface

$$\nabla f = \nabla(y-h(x,z)) = (-h_x\bar{a}_y + \bar{a}_y - h_z\bar{a}_z) = \bar{n}|\nabla f \tag{4.9}$$

and

$$h_x = \partial h/\partial x \text{ and } h_z = \partial h/\partial z \quad . \tag{4.10}$$

The conditional density can be expressed as

$$p(U|\bar{n}) = P_2(\bar{n}^f,\bar{n}^i|\bar{n})\delta(U-1)+[1-P_2(\bar{n}^f,\bar{n}^i|\bar{n})]\delta(U) \tag{4.11}$$

in which $\delta(U)$ is the Dirac delta function and $P_2(\bar{n}^f,\bar{n}^i|\bar{n})$ is the probability that a point on the rough surface will be both illuminated by the source and visible at the observation point, given the value of the gradient $\bar{n}(h_x,h_z)$.

On assuming that

$$k_o^2\langle h^2\rangle = k_o^2\sigma_o^2 \ll 1,\ \bar{n}\cdot\bar{a}_y \simeq 1,\ P_2(\bar{n}^i,\bar{n}^f|\bar{n}) = 1 \tag{4.12}$$

the full wave solutions for the scattering cross section (Bahar, 1981b) can be shown to reduce to the perturbation solution $\langle\sigma_o^{PQ}\rangle$ (Rice, 1951; Barrick, 1970)

$$\langle\sigma_o^{PQ}\rangle = 4\pi\ k_o^4 W(v_x,v_z)|\cos\theta_o^f\ \cos\theta_o^i\ \alpha^{PQ}|^2 \tag{4.13}$$

in which $\theta_o^i$ and $\theta_o^f$ are the angles of incidence and scatter with respect to the reference plane (y=0),

$$W(v_x,v_z) = \frac{1}{\pi^2}\int \exp(iv_x x_d + iv_z z_d)\ h(x,z)h'(x',z')\ dx_d dz_d \qquad (4.14)$$

is the spectral density of the rough surface height (Barrick, 1970; Ishimaru, 1978). It is assumed, in this work, that the rough surface is statistically homogeneous and isotropic, thus, the surface height correlation function $\langle h(x,z)h'(x',z')\rangle$ depends only on distances in the mean plane where $\langle h\rangle = 0$

$$\bar{r}_m = (x-x')\bar{a}_x + (z-z')\bar{a}_z = x_d\bar{a} + z_d\bar{a}_z \qquad (4.15)$$

Furthermore,

$$|2\cos\theta_o^f \cos\theta_o^i\ \alpha^{PQ}|^2 = \left|(\cos\theta_o^f + \cos\theta_o^i)D^{PQ}\right|^2_{\bar{n}\to\bar{a}_y} \qquad (4.16)$$

Thus to obtain the perturbation solution the unit vector $\bar{n}$ (normal to the rough surface) is replaced by $\bar{a}_y$ in the full wave expression for the scattering coefficient $D^{PQ}$ (Bahar, 1981b).

The Physical Optics solution for the scattering cross section $\langle\sigma_\infty^{PQ}\rangle$ can also be derived directly from the full wave solutions. Thus (Bahar, 1981c) for P=V or H and Q=H,V (P≠Q)

$$\langle\sigma_\infty^{PP}\rangle = \frac{k_o^2}{\pi}\left(\frac{2\cos\psi^{fs}\cos\psi^{is}\cos^2\theta_o^s}{\cos\theta_o^f + \cos\theta_o^i}\right)^2$$

$$\cdot|R_{10}^{Ps} + R_{10}^{Qs}\tan\psi^{fs}\tan\psi^{is}|^2\ P_2(\bar{n}^f,\bar{n}^i|\bar{n}_s)\mathcal{J} \qquad (4.17a)$$

and

$$\langle\sigma_\infty^{PQ}\rangle \simeq \frac{k_o^2}{\pi}\left(\frac{2\cos\psi^{fs}\cos\psi^{is}\cos^2\theta_o^s}{\cos\theta_o^f + \cos\theta_o^i}\right)^2$$

$$\cdot|R_{10}^{Ps}\tan\psi^{is} - R_{10}^{Qs}\tan\psi^{fs}|^2\ P_2(\bar{n}^f,\bar{n}^i|\bar{n}_s)\mathcal{J} \qquad (4.17b)$$

in which $\psi^{is}$ is the angle between the plane of incidence associated with the reference plane (y=0) and the local plane of incidence (associated with the local tangent plane) evaluated at the specular points where

$$\bar{n} + \bar{n}_s = \frac{\bar{n}^f - \bar{n}^i}{|\bar{n}^f - \bar{n}^i|} = \bar{v}/v \tag{4.18}$$

Similarly $\psi^{fs}$ is the angle between the reference and local scatter plane evaluated at the specular point. In (4.17) the Fresnel reflection coefficients $R_{10}^{Ps}$ and $P_2(\bar{n},\bar{n}^i|\bar{n})$, (4.12) are evaluated at $\theta_o^s$ the angle of incidence at the specular points. The integral $\mathcal{I}$ is given by

$$\mathcal{I} = \int_{-\infty}^{\infty} [\chi_2(v_y,-v_y) - |\chi(v_y)|^2]\exp[iv_x x_d + iv_z z_d]dx_d dz_d \tag{4.19}$$

in which $\chi(v_y)$ is the characteristic function (4.5) and $\chi_2(v_y,-v_y)$ is the joint characteristic function (transform of the height probability density $p(h,h')$)

$$\chi_2(v_y,-v_y) = \int_{-\infty}^{\infty} \exp[iv_y(h-h')]p(h,h')dhdh' \quad . \tag{4.20}$$

The full wave approach has also been applied to rough surfaces with two or more roughness scales. Thus, if it is assumed that the composite surface can be represented by the superposition of N uncorrelated surface heights $h_n (n=1,2...N)$ and that the principal feature that distinguishes the individual surface heights from each other is their respective correlation distances $\ell_n$ such that

$$\ell_1 << \ell_{12} << \ell_2 << \ell_{23} << \ell_3 \ldots \ell_{N-1} << \ell_{N-1,N} << \ell_N \tag{4.21}$$

the scattering cross section can be expressed as a weighted sum of the individual scattering cross sections $\langle\sigma^{PQ}\rangle_n$ (Bahar, 1981c)

$$\langle\sigma^{PQ}\rangle = \sum_{n=1}^{N} w_n \langle\sigma^{PQ}\rangle_n \tag{4.22}$$

in which $\langle\sigma^{PQ}\rangle_n$ is given by (4.3) with h,h' and $\chi$ replaced by $h_n, h_n'$ and $\chi^n$ (the characteristic function for surface $h_n$). The weighting function is given by the product

$$w_n = \prod_{m=1}^{n} |\chi^{m-1}|^2 \tag{4.23}$$

in which $\chi^{o} \equiv 1$. Since $|\chi^{n}| \leq 1$ the weighting function is in general less than unity.

For a two scale composite surface such that

$$k_o^2\langle h_1^2\rangle \ll 1,\ k_o^2\langle h_2^2\rangle \gg 1 \tag{4.24a}$$

$$\bar{n} \simeq \bar{a}_y\ ,\ \ell_1 \ll \ell_{12} \ll \ell_2 \tag{4.24b}$$

the scattering cross section is given by (Bahar, 1981c)

$$\langle\sigma^{PQ}\rangle \simeq \langle\sigma^{PQ}\rangle_1 + \langle\sigma^{PQ}\rangle_2 \tag{4.25}$$

In (4.25) $\langle\sigma^{PQ}\rangle_1$ is given by the perturbation solution (4.13) which accounts for Bragg scatter. Furthermore, $\langle\sigma^{PQ}\rangle_2$ is given by the Physical Optics solution (4.17). For a surface with a Gaussian distribution $\mathcal{J}$ in (4.17) is given by

$$\mathcal{J} = \left(\frac{2\pi}{k_o(\cos\theta_o^f + \cos\theta_o^i)}\right)^2 p(h_{xsp}, h_{zsp}) \tag{4.26}$$

in which the joint probability density function for the slopes at the specular points is $p(h_{xsp}, h_{zsp})$ (Bahar, 1981c). Thus the full waves solution (4.3) accounts for both Bragg scatter as well as specular-point scatter.

In Section 4.4 the full wave approach is applied to composite rough surfaces with moderate slopes ($h_x \simeq 1$, $h_z \simeq 1$).

## 4.4 Scattering Cross Sections for Composite Rough Surfaces with Moderate Slopes

Consider here a composite surface with two statistically independent roughness scales such that the radius vector to the surface is

$$\bar{r} = \bar{r}_F(x, h_F, z) + \bar{r}_R(\bar{h}_R) \tag{4.27a}$$

in which

$$k_o^2\langle h_F^2\rangle \gg 1,\ \frac{\partial h_F}{\partial x} = h_{Fx} \simeq h_x\ ,\ \frac{\partial h_F}{\partial z} = h_{Fz} \simeq h_z, \tag{4.27b}$$

$$\bar{r}_R(\bar{h}_R) = \bar{h}_R\ \bar{n} \tag{4.27c}$$

and

$$k_o^2\langle \bar{h}_R^2\rangle << 1, \quad \left|\frac{\partial \bar{h}_R}{\partial x}\right| = |\bar{h}_{Rx}| << 1, \quad \left|\frac{\partial \bar{h}_R}{\partial z}\right| = |\bar{h}_{Rz}| << 1 , \tag{4.27d}$$

In view of (4.27) and (4.9) the normal to the surface is

$$\bar{n} = \frac{\nabla(y-h)}{|\nabla(y-h)|} \simeq \frac{\nabla(y-h_F)}{|\nabla(y-h_F)|} = \bar{n}_F \tag{4.28}$$

However, unlike the composite surface considered in Section 4.3, the variances of the slopes are not assumed to be small. In addition, it is assumed that the radii of curvature of the surface $h_F$ (and therefore h) is large compared to the electromagnetic wavelength $\lambda$. Thus the full wave expression for the scattering cross section for the composite surface is (4.3)

$$\langle\sigma^{PQ}\rangle = \frac{k_o^2}{\pi A_y}\int\left[\langle S^{PQ}\exp[iv_y(h_F - h_F')]\chi_2^R\rangle - \left|\left\langle\frac{D^{PQ}U}{\bar{n}\cdot\bar{a}_y}\right\rangle\chi^F\chi^R\right|^2\right]$$

$$\cdot\exp[iv_x(x-x') + iv_z(z-z')]dxdz\, dx'dz' \tag{4.29}$$

In (4.29) $\chi^F$ and $\chi^R$ are the characteristic functions for surfaces $h_F$ and $\bar{h}_R$ respectively and $\chi_2^R$ is the joint characteristic function for the surface $\bar{h}_R$. In view of the above assumptions for the small scale surface*

$$\chi^R(v_{\bar{y}}) = \langle\exp[iv_{\bar{y}}\bar{h}_R]\rangle \simeq 1 - \frac{1}{2}v_{\bar{y}}^2\langle\bar{h}_R^2\rangle \simeq 1, \quad v_{\bar{y}} = \bar{n}\cdot\bar{v} \tag{4.30a}$$

$$\chi_2^R(v_{\bar{y}},-v_{\bar{y}}) = \langle\exp\, iv_{\bar{y}}(\bar{h}_R-\bar{h}_R')\rangle \simeq 1 - \frac{1}{2}v_{\bar{y}}^2(\langle\bar{h}_R^2\rangle + \langle\bar{h}_R'^2\rangle) + v_{\bar{y}}^2\langle\bar{h}_R\bar{h}_R'\rangle$$

$$\simeq |\chi^R|^2 + v_{\bar{y}}^2\langle\bar{h}_R\bar{h}_R'\rangle \tag{4.30b}$$

and

$$|\chi^F(v_y)|^2 << 1 \tag{4.31}$$

Following the analytical (Physical Optics) procedures presented by Sancer (1969) for the large scale surface $h_F$

- - - - - - - - - -

*The surface $\bar{h}_R$ which consists of the small scale spectral components cannot be treated as the large scale surface $h_F$ since it does not satisfy the radii of curvature criteria.

$$S^{PQ} \to \left|\frac{D^{PQ}(\bar{r})U(\bar{r})}{\bar{n}\cdot\bar{a}_y}\right|^2 \tag{4.32}$$

and

$$\exp iv_y(h_F - h_F') \to \exp iv_y(h_x x_d + h_z z_d) \tag{4.33}$$

Thus (4.29) can be expressed as follows:

$$\langle\sigma^{PQ}\rangle = \frac{k_o^2}{\pi A_y}\int \left\langle\left|\frac{D^{PQ}U}{\bar{n}\cdot\bar{a}_y}\right|^2 \exp[iv_y(h_x x_d + h_z z_d)](|\chi^R|^2 + v_{\bar{y}}^2\, \bar{h}_R \bar{h}_R')\right\rangle$$

$$\cdot\exp[iv_x x_d + iv_z z_d]dx_d dz_d dx dz$$

$$= \langle\sigma^{PQ}\rangle_o + \langle\sigma^{PQ}\rangle_1 \tag{4.34}$$

in which

$$\langle\sigma^{PQ}\rangle_o = \frac{k_o^2}{\pi}\int \left\langle\left|\frac{D^{PQ}U}{\bar{n}\cdot\bar{a}_y}\right|^2 \exp[iv_y(h_x x_d + h_z z_d)]|\chi^R|^2\right\rangle$$

$$\cdot\exp[iv_x x_d + iv_z z_d]dx_d dz_d = |\chi^R|^2\langle\sigma_\infty^{PQ}\rangle \tag{4.35}$$

Thus $\langle\sigma_\infty^{PQ}\rangle_o$ is the Physical Optics solution $\langle\sigma_\infty^{PQ}\rangle$, (4.17) multiplied by $|\chi^R(\bar{v}\cdot\bar{n}_s)|^2 < 1$, the characteristic function for the small scale rough surface evaluated at $\bar{v}\cdot\bar{n}_s$ the projection of $\bar{v}$ on the normal at the specular point. Furthermore,

$$\langle\sigma^{PQ}\rangle_1 = \frac{k_o^2}{\pi}\left\langle\int\left|\frac{D^{PQ}Uv_{\bar{y}}}{\bar{n}\cdot\bar{a}_y}\right|^2 \bar{h}_R\bar{h}_R' \exp[i(v_x + v_y h_x)x_d + i(v_z + v_y h_z)z_d]dx_d dz_d\right\rangle \tag{4.36}$$

Thus $\langle\sigma^{PQ}\rangle_1$ reduces to the perturbation solution (4.13) when the surface slopes are small (Bahar, 1981b). For the case considered in this section the distribution of the large scale surface slopes has an effect on the component of the scattering cross section due to the small scale roughness $\langle\sigma^{PQ}\rangle_1$.

Note that the small scale roughness $\bar{h}_R$ is a perturbation about the large scale roughness and $\bar{h}_R$ is the perpendicular distance from the unperturbed (filtered) surface $y = h_F(x, h_F, z)$ to the perturbed surface (4.27) (Burrows, 1973). Thus the expectation of $\bar{h}_R(x,z)\bar{h}_R'(x',z')$ is a function of distances measured along the plane tangent to the

surface $y - h_F(x,z) = 0$ (Wright, 1966; Valenzuela,1968) and not distances measured along the mean plane $|\bar{r}_m|$ (4.15). To facilitate the evaluation of (4.36) the integrand is expressed in terms of the unit vectors $\bar{n}_1$ $\bar{n}_2$ and $\bar{n}_3$ and variables $\bar{x},\bar{y},\bar{z}$ of the local coordinate system associated with the local tangent plane (Bahar, 1981a; Bahar, 1981b). The unit vectors $\bar{n}_1,\bar{n}_2,\bar{n}_3$ can be chosen as follows (see Fig. 4.2)

$$\bar{n}_1 = (\bar{n} \times \bar{a}_3)/|\bar{n} \times \bar{a}_3|,\ \bar{n}_2 = \bar{n},\ \bar{n}_3 = \bar{n}_1 \times \bar{n} \tag{4.37a}$$

in which

$$\bar{a}_1 = \bar{a}_x,\ \bar{a}_2 = \bar{a}_y,\ \bar{a}_3 = \bar{a}_z \tag{4.37b}$$

and the relationship between the local coordinate system $X(\bar{x}^1=\bar{x},\bar{x}^2=\bar{y},\bar{x}^3=\bar{z})$ and the reference coordinate system $X(x^1=x,x^2=y,x^3=z)$ is

$$\bar{x}^i = a_{ij}x^j \text{ (sum on } j=1,2,3) \tag{4.38a}$$

in which

$$a_{ij} = \bar{n}_i \cdot \bar{a}_j \quad i,j=1,2,3 \tag{4.38b}$$

Thus in (4.36)

$$(v_x+v_y h_x)x_d+(v_z+v_y h_z)z_d = v_{\bar{x}}\,\bar{x}_d + v_{\bar{z}}\,\bar{z}_d \tag{4.39a}$$

in which $v_{\bar{x}}$, $v_{\bar{y}}$, $v_{\bar{z}}$ are the components of $\bar{v}$ (4.4) in the local coordinate system

$$v_{\bar{x}} = \bar{v}\cdot\bar{n}_1,\ v_{\bar{y}} = \bar{v}\cdot\bar{n}_2,\ v_{\bar{z}} = \bar{v}\cdot\bar{n}_3 \tag{4.39b}$$

Furthermore (see Fig. 4.3)

$$\bar{x}_d = \bar{x}-\bar{x}' \quad ,\ \bar{z}_d = \bar{z}-\bar{z}' \tag{4.40a}$$

and

$$\frac{dx_d dz_d}{\bar{n}\cdot\bar{a}_y} = d\bar{x}_d\, d\bar{z}_d \tag{4.40b}$$

Therefore (4.36) can be expressed as

$$\langle\sigma^{PQ}\rangle_1 = \frac{k_o^2}{\pi}\left\langle\int\frac{|D^{PQ}Uv_{\bar{y}}|^2}{\bar{n}\cdot\bar{a}_y}\,\bar{h}_R\bar{h}_R'\,\exp[iv_{\bar{x}}\bar{x}_d+ iv_{\bar{z}}\bar{z}_d]d\bar{x}_d\, d\bar{z}_d\right\rangle \tag{4.41}$$

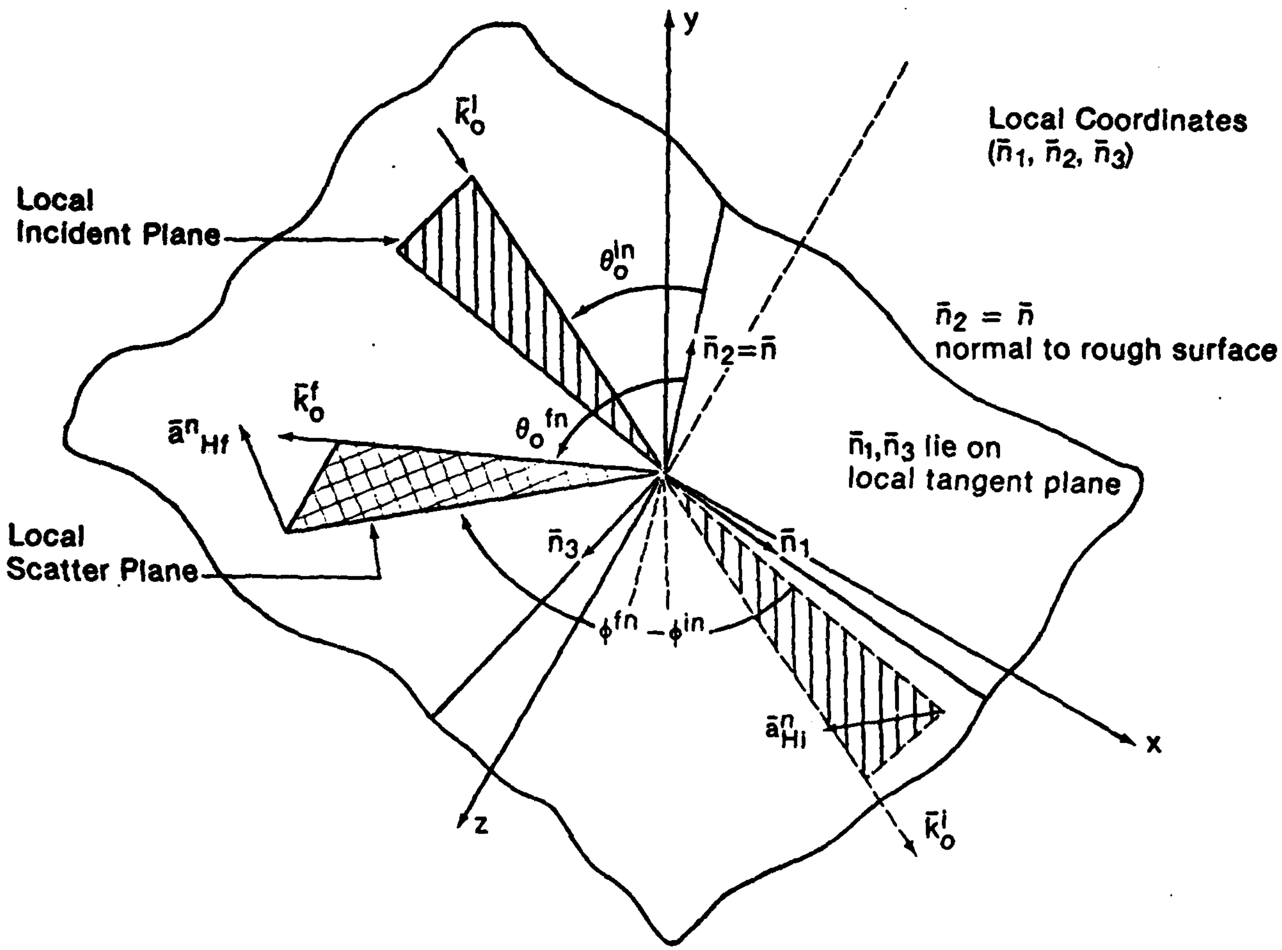



Figure 4.2 Local planes of incidence and scatter and the local coordinate system with unit vectors $\bar{n}_1$, $\bar{n}_2$ and $\bar{n}_3$.

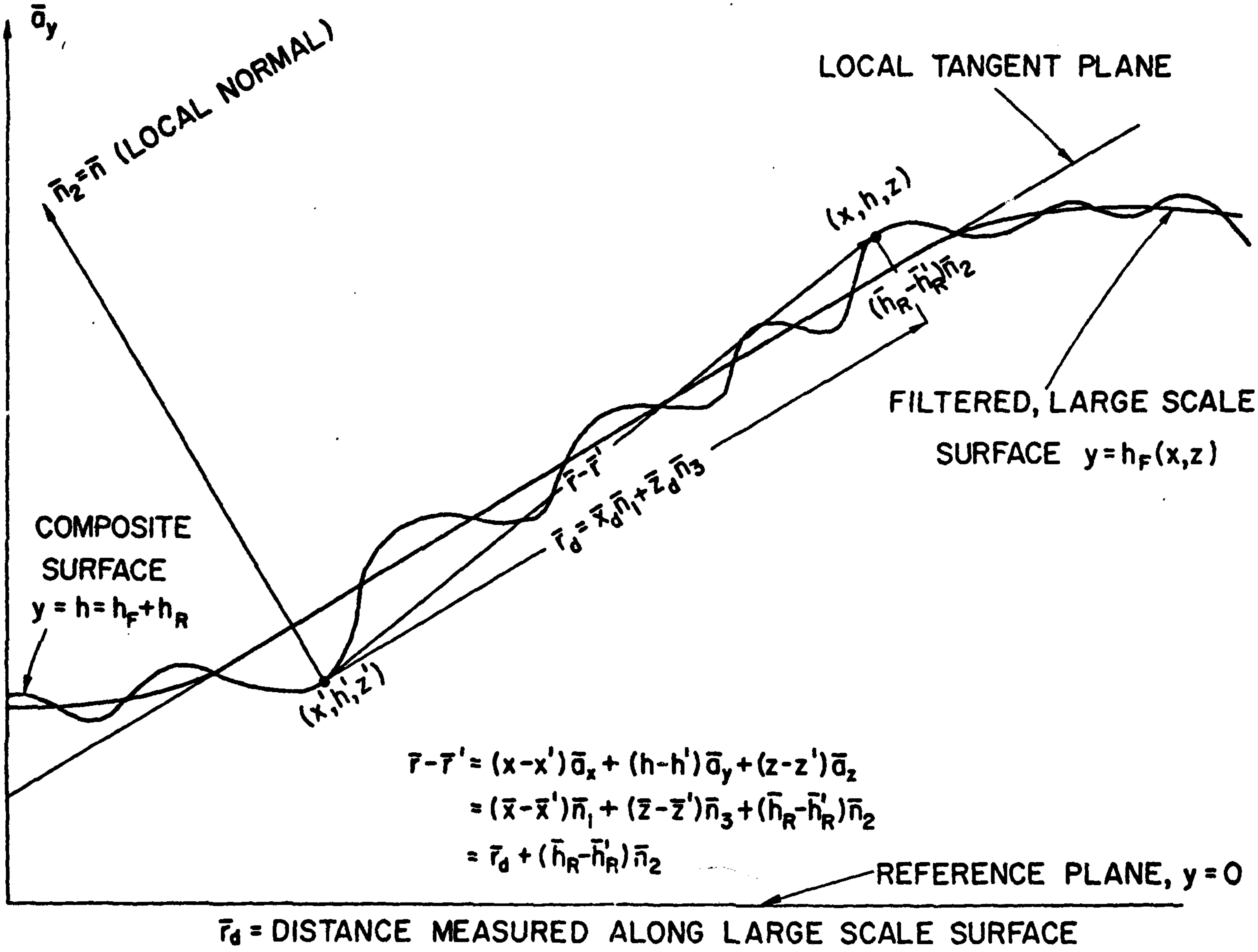



Figure 4.3 Rough surface decomposed into large and small scale surfaces.

Furthermore assuming that the statistics of the small scale surface $\bar{h}_R$ are independent of the slopes of the large scale surface $h_F$* (Valenzuela, 1968, Brown, 1978) (see Fig. 4.3)

$$\frac{1}{\pi^2}\int\langle\bar{h}_R(\bar{x},\bar{z})\bar{h}_R'(\bar{x}',\bar{z}')\rangle\exp(iv_{\bar{x}}\bar{x}_d+iv_{\bar{z}}\bar{z}_d)d\bar{x}_d\, d\bar{z}_d = W(v_{\bar{x}},v_{\bar{z}}) \tag{4.42}$$

in which the dependent variables of the spectral density of the small scale roughness $v_{\bar{x}}$ and $v_{\bar{z}}$ are the orthogonal components of $\bar{v}$ in the local tangent plane. Thus $W(v_{\bar{x}},v_{\bar{z}})$ depends not only on the direction of the incident and scattered waves but also on the normal, $\bar{n}$, to the rough surface. Substitute (4.42) into (4.41) to get

$$\langle\sigma^{PQ}\rangle_1 = \pi k_o^2\int\frac{|D^{PQ}Uv_{\bar{y}}|^2}{\bar{n}\cdot\bar{a}_y}W(v_{\bar{x}},v_{\bar{z}})p(\bar{n})p(U|\bar{n})dh_x dh_z dU \tag{4.43}$$

and substitute (4.11) for $P(U|\bar{n})$ unto (4.43) and integrate with respect to U to get

$$\langle\sigma^{PQ}\rangle_1 = \pi k_o^2\int\frac{|D^{PQ}v_{\bar{y}}|^2}{\bar{n}\cdot\bar{a}_y}W(v_{\bar{x}},v_{\bar{z}})P_2(\bar{n}^f,\bar{n}^i|\bar{n})p(\bar{n})dh_x dh_z \tag{4.44}$$

Note that the conditional probability $P_2(\bar{n}^f,\bar{n}^i|\bar{n})$ for arbitrary slope $\bar{n}(h_x,h_z)$ can be expressed in terms of the conditional probability $P_2(\bar{n}^f,\bar{n}^i|\bar{n}_s)$ for the specular points $(\bar{n}\to\bar{n}_s)$ as follows (Smith, 1967; Brown, 1980)

$$P_2(\bar{n}^f,\bar{n}^i|\bar{n}) = S(\bar{n}\cdot\bar{n}^f)S(-\bar{n}\cdot\bar{n}^i)P_2(\bar{n}^f,\bar{n}^i|\bar{n}_s) \tag{4.45}$$

in which $S(\alpha)$ is the unit step function. The arguments of $S$, $-\bar{n}\cdot\bar{n}^i$ and $\bar{n}\cdot\bar{n}^f$ vanish at points of the rough surface where the incident and scatter wave normals, $\bar{n}^i$ and $\bar{n}^f$ are tangent to the surface.

In order to interpret the results (4.44) assume that the surface $h_F$ is normal to the constant vector $\bar{n}_o$

---

*This assumption of statistical independence of the small scale ... $\bar{h}_R$ and slopes restricts the class of rough surfaces assumed in this work.

$$\bar{n} \to \bar{n}_o = n_{ox}\bar{a}_x + n_{oy}\bar{a}_y + n_{oz}\bar{a}_z \tag{4.46a}$$

The equation for this surface is

$$\bar{n}_o \cdot \bar{r} = n_{ox}\, x + n_{oy}\, y + n_{oz}\, z = 0 \tag{4.46b}$$

thus for $y = h_F(x,a)$

$$h_F = -(n_{ox}\, x + n_{oz}\, z)/n_{oy} \tag{4.46c}$$

Furthermore

$$p(\bar{n}) = \delta(h_x + n_{ox}|n_{oy})\delta(h_z + n_{oz}|n_{oy}) = \delta(\bar{n}-\bar{n}_o) \tag{4.47}$$

Assuming that $\bar{n}\cdot\bar{n}^f > 0$ and $-\bar{n}\cdot\bar{n}^i > 0$ (the plane $\bar{n}_o\cdot\bar{r} = 0$ is visible and illuminated), the normalized scattering cross section $\langle\sigma^{PQ}\rangle_1$ (4.45) is given

$$\langle\sigma^{PQ}\rangle_1 = \pi k_o^2\left[\frac{W(v_{\bar{x}}, v_{\bar{z}})\,|D^{PQ}v_{\bar{y}}|^2}{\bar{n}_o\cdot\bar{a}_y}\right]_{\bar{n}=\bar{n}_o} \tag{4.48}$$

Thus the scattering cross section (4.48) for the inclined plane (4.46) is in complete agreement with the perturbation result (4.13) for arbitrary $\bar{n}_o$. It is equal to it for $\bar{n}_o = \bar{a}_y$ (Rice, 1951; Barrick, 1970). The term $\bar{n}_o\cdot\bar{a}_y$ appearing in the denominator of (4.48) is a consequence of normalization (4.2) (since $A_y$ is the projection of the surface A on the reference plane).

The integrand in (4.44) is therefore the contribution to the scattering cross section $\langle\sigma^{PQ}\rangle_1$ from "patches" of the rough surface that are normal to the vector $\bar{n}$ (Wright, 1968; Valenzuela, 1968).

In his review of theoretical treatments of the scattering of electromagnetic waves by rough surfaces, Valenzuela (1968) states that scattering models have been developed "that make it possible to apply available theories to surfaces that cannot be treated purely by perturbation and Physical Optics....in these models composite surface approximate the rough surface by 'patches' of slightly rough surfaces

that ride the large waves....the total scattered power from this composite surface is obtained by averaging over the distribution of slopes of the large waves." Valenzuela notes however, that "these composite models are mostly based on physical considerations and are able to explain features in radar cross-section data from the sea that no theory can." It is shown in this section that using the full wave approach, one can provide a self-consistent theoretical basis for scattering by composite surfaces without using a combination of Physical Optics and perturbation theories.

To compare this work with earlier analysis of the backscatter cross section (Valenzuela, 1968; Valenzuela, Laing and Daley, 1971) in which the integral (4.44) is expressed in terms of tilt angles $\psi$ and $\delta$ "in" and "perpendicular" to the reference plane of incidence (normal to $\bar{n}^i \times \bar{a}_y$) rather than in terms of the slopes $h_x$ and $h_z$, the normal $\bar{n}$ to the rough surface is expressed as follows for $\bar{n}^i$ in the (x,y) plane

$$\begin{aligned} \bar{n} &= \sin\psi \cos\delta \, \bar{a}_x + \cos\psi \cos\delta \, \bar{a}_y + \sin\delta \, \bar{a}_z \\ &= (- h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z)/(1 + h_x^2 + h_z^2)^{1/2} \end{aligned} \tag{4.49}$$

In the work by Valenzuela, et al. (1971), shadowing had been ignored, however the arguments of $W(v_{\bar{x}}, v_{\bar{z}})$ in their work is in agreement with (4.44).

The result (4.44') is also in agreement with the expression derived recently by Brown (1978) (as corrected in Brown, 1980) provided that the slope of the large scale rough surface is very small. The differences between these two solutions are primarily due to the assumption in Brown's work that $\bar{h}_R$ ($\zeta_s$ in Brown's notation) is the distance from the unperturbed (filtered) surface to the perturbed surface, measured along a line perpendicular to the reference plane (y=0) rather than perpendicular to the unperturbed surface. This is contrary to the original analysis of Burrows (1973) upon which Brown's analysis (1978) is based. Furthermore in this work, the expectation of $\bar{h}_R \bar{h}_R'$ is considered to be dependent on distances $\bar{r}_d$ measured

along the plane tangent to the surface $y=h_1(x,z)$, (i.e., the surface normal to $\bar{n}$) where $\bar{r}_d = \bar{x}_d\ \bar{n}_1 + \bar{z}_d\ \bar{n}_3$, instead of distances measured along the mean (reference) plane $\bar{r}_m = x_d\ \bar{a}_x + z_d\ \bar{a}_z$ (4.15) as assumed by Brown (see Fig.4.3).This feature makes the full wave solutions invariant to coordinate transformations (4.48). In his analysis, Brown (1978, 1980) accounts for shadowing and uses both Physical Optics and Burrows (1973) perturbation theories to derive his results in terms of a two dimensional convolution of transforms.

## 4.5 Concluding Remarks

Composite models of random rough surfaces with moderate mean square slopes are analyzed using the full wave approach. The full wave approach is shown to account for specular point scattering and Bragg scattering in a self-consistent manner without introducing Physical Optics and perturbation theories. Thus using the full wave approach, it is not necessary to specify the wavenumber $k_d$ where the rough surface spectral splitting is assumed to occur (Brown, 1978, 1980). However, decomposition of the surface into a large scale surface $h_\ell$ and small scale surface $h_s$ enhances one's physical insight and facilitates the numerical evaluation of the scattering cross sections. The wavenumber $k_d$ is used to distinguish between the large scale rough surface, $h_F$, and the small scale rough surface, $\bar{h}_R$. The solutions derived in this paper are compared with solutions derived earlier "mostly based on physical considerations" (Valenzuela, 1968). They are also shown to be in agreement with Brown's solution provided that the mean square slope of the rough surface is very small (Brown, 1978, 1980). Brown uses a combination of Physical Optics (Beckmann, 1968) and Burrows (1967)

perturbation theories to develop his solution. The differences between Brown's solution and the full wave solution (4.44) are primarily due to the assumption in Brown's work that $\bar{h}_R$ is the distance, from the unperturbed surface to the perturbed surface, measured along a line perpendicular to the reference plane (y=0) rather than perpendicular to the unperturbed (filtered) surface. This is contrary to the original analysis of Burrows (1973) upon which Brown's (1978) analysis is based. Furthermore, Brown assumes that the autocorrelation function for the small scale surface height depends upon distances measured in the mean plane rather than distances along the plane tangent to the large scale surface as assumed here and implicitly by Valenzuela (1968).

The contribution to the scattering cross section due to Bragg scattering (associated with the small scale surface) is given by an integral over the slopes of the rough surface. This contribution is proportional to $k_o^4$ and depends on the polarization (4.44). It is shown to be in complete agreement with perturbation theory (Rice, 1951; Barrick, 1970; Barrick and Peake, 1968). The integrand in this expression is proportional to the small scale surface height spectral density $W(v_{\bar{x}},v_{\bar{z}})$ whose arguments are the components of $\bar{v} = k_o(\bar{n}^f-\bar{n}^i)$ in the local tangent plane. Thus $W(v_{\bar{x}},v_{\bar{z}})$ is also a function of slopes. The contribution to the scattering cross section due to specular point scattering (associated with the large scale surface) is independent of $k_o$ and polarization.

Using the full wave approach the specification of $k_d$ (where spectral splitting is assumed to occur is not restricted by the perturbation condition, $\beta = 4k_o^2\langle h_s^2\rangle \ll 1$. Thus it can be applied to more general models of rough surfaces for which a combination of solutions based on perturbation and Physical Optics are not suitable. The specifications of $k_d$ and the sensitivity of the cross sections $\langle\sigma^{PQ}\rangle$ to variations in $k_d$ are subjects of future investigations.

## 4.6 References


1. Bahar, E., "Full Wave Solutions for the Depolarization of the Scattered Radiation Fields by Rough Surfaces of Arbitrary Slope," IEEE Transactions on Antennas and Propagation, AP-29(3), pp. 443-454, 1981a.
2. Bahar, E., "Scattering Cross Sections from Random Rough Surfaces--Full Wave Analysis," Radio Science, 16(3), pp. 331-341, 1981b.
3. Bahar, E., "Scattering Cross Sections for Composite Random Surfaces--Full Wave Analysis," Radio Science, 10(5), in press, 1981c.
4. Barrick, D. E., Rough Surfaces, in Radar Cross Section Handbook, (Ch. 9), Plenum Press, New York, 1970.
5. Barrick, D. E. and W. H. Peake, "A Review of Scattering from Surfaces with Different Roughness Scales," Radio Science, 3(8), pp. 865-868, 1968.
6. Beckmann, P., The Depolarization of Electromagnetic Waves, Golem Press, Boulder, Colorado, 1968.
7. Brown, G. S., "Backscattering from a Gaussian-Distributed Perfectly Conducting Rough Surface," IEEE Transactions on Antennas and Propagation, AP-26(3), pp. 472-482, 1978.
8. Brown, G. S., "Correction to Backscattering from a Gaussian-Distributed Perfectly Conducting Rough Surface," IEEE Transactions on Antennas and Propagation, AP-28(6), pp. 943-946, 1980.
9. Burrows, M. L., "On the Composite Model for Rough Surface Scattering," IEEE Transactions on Antennas and Propagation, AP-21(2), pp. 241-243, 1967.
10. Hagfors, T., "Relationship of Geometric Optics and Autocorrelation Approach to the Analysis of Lunar and Planetary Radar," Journal of Geophysical Research, 71, pp. 379-383, 1966.

11. Ishimaru, A., Wave Propagation and Scattering in Random Media in Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing, (Vol. 2, Ch. 21), Academic Press, New York, 1978.

12. Rice, S. O., "Reflection of Electromagnetic Waves from a Slightly Rough Surface," Communication of Pure and Applied Math, 4, pp. 351-378, 1951.

13. Sancer, M. I., "Shadow-Corrected Electromagnetic Scattering from a Randomly Rough Surface," IEEE Transactions on Antennas and Propagation, AP-17(5), pp. 577-585, 1969.

14. Semyonov, B. J., "Approximate Computation of Scattering of Electromagnetic Waves by Rough Surface Contours (translation)," Radio Engnr. Electron. Phys., 11(8), pp. 1179-1187, 1966.

15. Smith, B. G., "Geometrical Shadowing of a Randomly Rough Surface," IEEE Transactions on Antennas and Propagation, AP-15(5), pp. 668-671, 1967.

16. Tyler, G. L., "Wavelength Dependence in Radio-Wave Scattering and Specular Point Theory," Radio Science, 11(1), pp. 83-91, 1976.

17. Wright, J. W., "Backscattering from Capillary Waves with Application to Sea Clutter," IEEE Transactions on Antennas and Propagation, AP-14(6), pp. 749-754, 1966.

18. Wright, J. W., "A New Model for Sea Clutter," IEEE Transactions on Antennas and Propagation, AP-16(2), pp. 217-223, 1968.

19. Valenzuela, G. R., "Scattering of Electromagnetic Waves from a Tilted Slightly Rough Surface," Radio Science, 3(11), pp. 1051-1066, 1968.

20. Valenzuela, G. R., M. B. Laing and J. C. Daley, "Ocean Spectra for the High Frequency Waves as Determined from Airborne Radar Measurements," Journal of Marine Research, 29(2), pp. 69-84, 1971.

# 5.0 SUMMARY OF RESEARCH TO BE CONDUCTED DURING SECOND YEAR OF CONTRACT

## 5.1 Analysis

In order to account for both specular point scattering as well as Bragg scattering, a two scale—composite model—of the rough surface is generally used. The specular point scattering is accounted for through the use of Physical Optics theory, while perturbation theory is used to account for Bragg scattering. The principal difficulty with this perturbed-Physical Optics approach to analyze composite models of rough surfaces lies in the specification of the wavenumber $k_d$ where the surface height spectral splitting is assumed to occur. On one hand $k_d$ must be sufficiently small such that the radii of curvature of the large scale (filtered) surface is large enough to justify the application of Physical Optics theory to the filtered surface. On the other hand $k_d$ must be sufficiently large such that the mean square height of the small scale surface is small enough to justify the application of perturbation theory to this small scale surface. In general these two restrictions on the specification of $k_d$ may not be satisfied simultaneously. It is found using this perturbed-Physical Optics approach, that the computed value of the rough surface scattering cross section will critically depend upon the specified value of $k_d$. Using the Full Wave approach (which accounts for both specular point scattering as well as Bragg scattering in a self-consistent manner), the specification of the wavenumber $k_d$ (at which spectral splitting is assumed to occur), is not restricted by the conflicting considerations of Physical Optics and perturbation theories. Rather in this case $k_d$ is determined only by analytical and computational considerations. Thus, if $k_d$ is chosen judiciously the

computed value for the resulting scattering cross section should not depend upon the specific value of $k_d$ chosen for these computations. Instead $k_d$ is determined solely by the need to minimize the numerical computations. This aspect of the problem is being investigated in detail.

During the next year, the Full Wave approach will also be applied to non-Gaussian rough surfaces. Thus the following topics will be investigated in detail:

a. Statistical description of classes of non-Gaussian rough surfaces.
b. Shadow functions for non-Gaussian rough surfaces.
c. Like and cross polarized scattering cross sections for non-Gaussian rough surfaces.

Rough surfaces for which decorrelation implies statistical independence are of particular interest in our investigations.

## 5.2 Computer Program

The computer programs used to evaluate the scattering cross sections for composite rough surfaces are being modified to reflect the recent advances made in the analytical approach to this problem. The effect of the choice of $k_d$ on the numerical computations is being investigated. From the preliminary results it is shown that the numerical value for the scattering cross sections as determined by the recently modified computer programs, are not dependent on $k_d$ and that the most suitable choice for $k_d$ is such that $4k_o^2\langle\bar{h}_R^2\rangle \simeq 1$ ($k_o$ is the electromagnetic wavenumber and $\langle\bar{h}_R^2\rangle$ is the mean square of the small scale surface height). More work needs to be done to complete this phase of the investigation.